

Spatio-temporal InSAR phase denoising with ADMM applied to an Interferometric Phase Linking Framework

Elena Grosso^{1,2}, Frederic Brigue¹, Guillaume Ginolhac³, Arnaud Breloy²

¹ DEMR, ONERA, Université Paris-Saclay, 91120 Palaiseau, France, ,

² CNAM, Conservatoire National des Arts et Métiers, 75003 Paris, France ³ LISTIC (EA-3703),
Université Savoie Mont Blanc, 74000 Annecy, France

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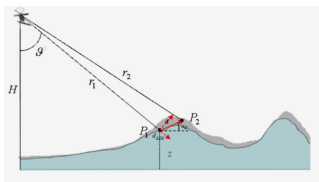


Académie Spatiale
d'Île-de-France

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Interferometric SAR: Classic 2-Pass InSAR

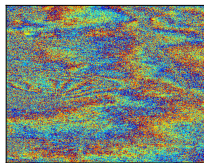
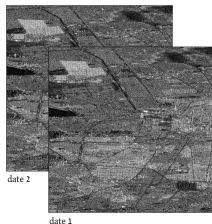


InSAR for displacement

$$\Delta\Phi = -\frac{4\pi}{\lambda} \cdot \frac{b_{\perp}}{r \sin \theta} z + \frac{4\pi}{\lambda} d_{LOS}$$

d_{LOS} : **terrain deformation** w.r.t. sensor's Line Of Sight (LOS)

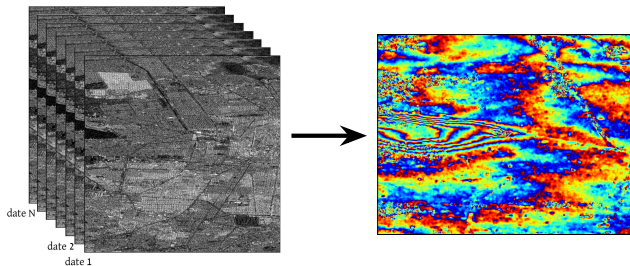
$$\Delta\Phi = \cancel{\Delta\Phi_{topo}} + \cancel{\Delta\Phi_{orb}} + \Delta\Phi_{displ} + \Delta\Phi_{atm} + \Delta\Phi_{noise}$$



2-Pass InSAR

→ classic methodology

Multi-temporal InSAR



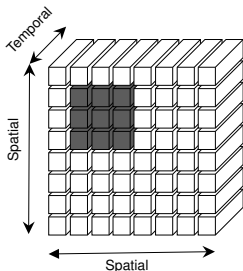
- displacement estimation over time
- Leverages the **redundancy** in time series

Requires new estimation methodology



Interferometric Phase Linking through Covariance Fitting method

Multi-InSAR Covariance Fitting model [2]



Statistical moments assumptions

on pixels $\mathbf{x} = [x^1, \dots, x^T] \in \mathbb{C}^T$

→ **Zero-mean** SAR images

$$\mathbb{E}[x^T] = 0, \forall t \in [1, T]$$

→ **Covariance between times** depends on phase differences

$$\mathbb{E}[x^k (x^l)^*] = \underbrace{v_{k,l} \sigma_k \sigma_l}_{\text{coh.} \times \text{var.}} \exp(j \underbrace{(\theta_k - \theta_l)}_{\text{phase diff.}})$$

- **Stack** of T images: $\mathbf{x} \in \mathbb{C}^T$

- **Patch** of n **multivariate** pixels

$\{\mathbf{x}_i\}_{i=1}^n \in \mathbb{C}^n$, with $\mathbf{x}_i \in \mathbb{C}^T$, $\forall i \in [1, n]$

→ similar scattering and statistical properties (**multivariate**)

Multi-InSAR Covariance Fitting model [2]

Covariance matrix follows **modulus-argument decomposition** structure:

with
$$\mathbb{E} [\mathbf{xx}^H] = \mathbf{\Sigma} = \text{mod}(\mathbf{\Sigma}) \odot \arg(\mathbf{\Sigma}) \triangleq \mathbf{\Psi} \odot (\mathbf{ww}^H)$$

$\mathbf{\Psi}$: matrix with coherency information

$\mathbf{w} = [e^{j\theta_1}, \dots, e^{j\theta_t}]$: phase-only vector

$\theta = [\theta_1, \dots, \theta_t]$: vector of SAR images phases ($\theta_1 = 0$ by convention).

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Optimization

matrix distance/divergence

minimize
 $\mathbf{w}$

subject to

$$d^2(\hat{\mathbf{\Sigma}}, \hat{\mathbf{\Psi}} \odot \mathbf{w}\mathbf{w}^H)$$

$$\mathbf{w} \in \mathbb{T}_t$$

endowed in Torus manifold

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In practice:

$$\hat{\mathbf{\Sigma}}: \text{plug-in CM} \rightarrow \text{SCM: } \mathbf{S} = \sum_{i=1}^n \mathbf{x}_i \mathbf{x}_i^H$$

$$d: \rightarrow \text{Frobenius dist. } \|\mathbf{S} - \text{mod}(\mathbf{S}) \odot \mathbf{w}\mathbf{w}^H\|_F^2$$

Spatio-temporal denoising: the framework

Add a **spatial regularisation** term to the IPL framework:

temporal phase denoising $\rightarrow \mathcal{L}(\mathcal{X}|\mathcal{W}) = \sum_{i,j}^{I,J} \|\mathbf{S}_{ij} - \text{mod}(\mathbf{S}_{ij}) \circ \mathbf{w}_{ij} \mathbf{w}_{ij}^H\|_F^2$

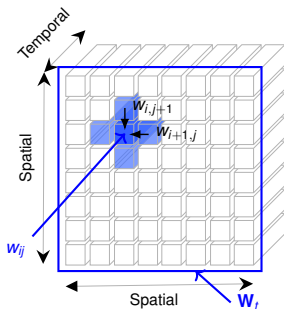
spatial regularisation $\rightarrow \phi(\mathcal{W}) = \sum_{t=1}^T h(\mathbf{W}_t)$

The resulting estimation problem becomes:

$$\begin{array}{ll} \underset{\mathcal{W}}{\text{minimize}} & \mathcal{L}(\mathcal{X}|\mathcal{W}) + \lambda \phi(\mathcal{W}) \\ \text{subject to} & \mathcal{W} \in \mathbb{T}_{I \times J \times T} \end{array}$$

where $\lambda > 0$

Spatial Regulariser: Total Variation



anisotropic (2D TV)

$$\text{TV}^{2\text{D}}(\mathbf{W}_t) = \sum_{i=1}^I \left(\sum_{j=1}^{J-1} \|w_{i,j\pm 1} - w_{i,j}\| \right) + \sum_{j=1}^J \left(\sum_{i=1}^{I-1} \|w_{i\pm 1,j} - w_{i,j}\| \right)$$

→ where $\mathbf{W}_t$ is the image of phases of the stack of time $t \in T$

The proximal mapping for the functions of each component of the formula is defined:

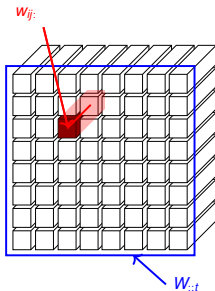
$$\text{prox}_{\epsilon G} w_{i,j}^k = \underset{w}{\operatorname{argmin}} \epsilon d_G^q(w_{i,j}^{\text{ref}_k}, w_{i,j}^k) + \frac{1}{2} d^2(w_{i,j}^k, w_{i,j}^{k-1}) \quad (1)$$

where G is each component of the objective function

Spatio-temporal denoising: the framework

temporal phase denoising $\rightarrow \mathcal{L}(\mathcal{X}|\mathcal{W}) = \sum_{i,j}^{I,J} \|\mathbf{S}_{ij} - \text{mod}(\mathbf{S}_{ij}) \circ \mathbf{w}_{ij} \mathbf{w}_{ij}^H\|_F^2$

spatial regularisation $\rightarrow \phi(\mathcal{W}) = \sum_{t=1}^T \text{TV}(\mathbf{W}_t)$



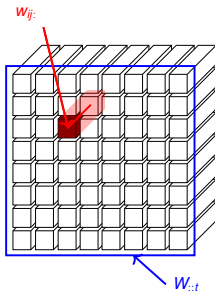
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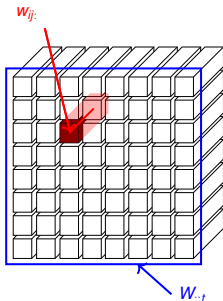
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PROBLEM: objective function is **not separable** $\rightarrow$ too big a problem to tackle

Problem Splitting

⇒ treat the datacube as two distinct parameters and **impose their equality**

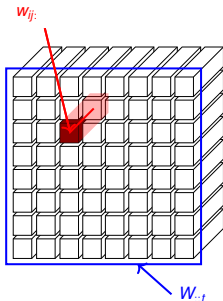


$$[\mathcal{W}]_{ij:} = \mathbf{w}_{ij:} \rightarrow \mathcal{W}^{\mathcal{L}}$$

$$[\mathcal{W}]_{::t} = \mathbf{W}_{::t} \rightarrow \mathcal{W}^{\phi}$$

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$$[\mathcal{W}]_{ij:} = \mathbf{w}_{ij:} \rightarrow \mathcal{W}^{\mathcal{L}}$$

$$[\mathcal{W}]_{::t} = \mathbf{W}_{::t} \rightarrow \mathcal{W}^{\phi}$$

Variable
splitting
reformulation:

$$\begin{array}{ll} \text{minimize} & \mathcal{L}(\mathcal{X} | \mathcal{W}^{\mathcal{L}}) + \lambda \phi(\mathcal{W}^{\phi}) \\ \mathcal{W}^{\mathcal{L}}, \mathcal{W}^{\phi} & \end{array}$$

$$\text{subject to } \mathcal{W}^{\mathcal{L}} \in \mathbb{T}_{I \times J \times T}$$

$$\mathcal{W}^{\phi} \in \mathbb{T}_{I \times J \times T}$$

$$\mathcal{W}^{\mathcal{L}} = \mathcal{W}^{\phi}$$

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Augmented Lagrangian Reformulation:

$$\begin{aligned} \mathcal{L}(\mathbf{w}^{\mathcal{L}}, \mathbf{w}^{\phi}, \Gamma) &= \mathcal{L}(\mathbf{x} | \mathbf{w}^{\mathcal{L}}) + \lambda \phi(\mathbf{w}^{\phi}) + \text{AL}^{\Gamma}(\mathbf{w}^{\mathcal{L}}, \mathbf{w}^{\phi}) \\ \text{AL}^{\Gamma}(\mathbf{w}^{\mathcal{L}}, \mathbf{w}^{\phi}) &= \gamma \|\mathbf{w}^{\mathcal{L}} - \mathbf{w}^{\phi}\|_F^2 + \langle \Gamma, \mathbf{w}^{\mathcal{L}} - \mathbf{w}^{\phi} \rangle \end{aligned}$$

where $\gamma \in \mathbb{R}^+$.

ADMM Steps

The objective function is minimised alternatively for each variable while keeping the others fixed.

3 Sequential Steps:

$\mathcal{W}^{\mathcal{L}}$ -Update $\rightarrow$ **temporal** component optimisation

$$\underset{\mathcal{W}^{\mathcal{L}} \in \mathbb{T}_{i \times j \times t}}{\text{minimize}} \quad \mathcal{L}(\mathcal{X} | \mathcal{W}^{\mathcal{L}}) + \text{AL}^{\Gamma}(\mathcal{W}^{\mathcal{L}}, \mathcal{W}^{\phi})$$

$\rightarrow$ solved for each pixel i, j

$\mathcal{W}^{\phi}$ -Update $\rightarrow$ **spatial** component optimisation

$$\underset{\mathcal{W}^{\phi}}{\text{minimize}} \quad \lambda \phi(\mathcal{W}^{\phi}) + \text{AL}^{\Gamma}(\mathcal{W}^{\mathcal{L}}, \mathcal{W}^{\phi})$$

$\rightarrow$ solved for each image t

Γ -Update $\rightarrow$ maintains constraint $\mathcal{W}^{\mathcal{L}} = \mathcal{W}^{\phi}$

$$\Gamma \leftarrow \Gamma + 2\gamma(\mathcal{W}^{\mathcal{L}} - \mathcal{W}^{\phi})$$

First Results Comparisons: Mexico City, Sentinel-1

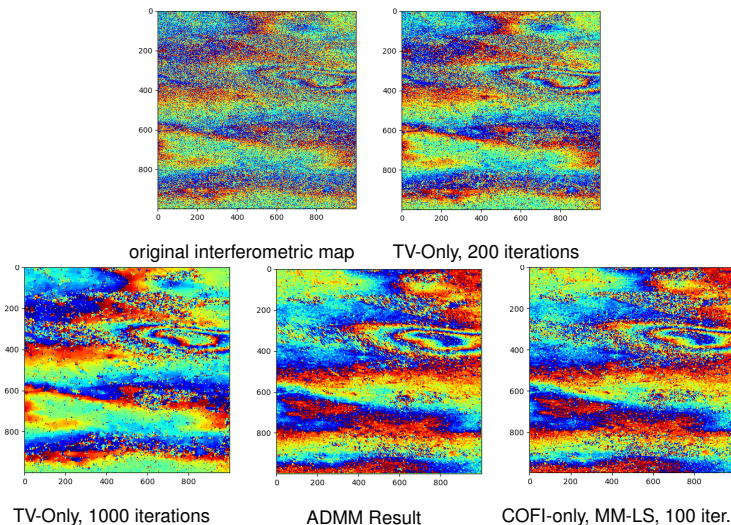


Figure: ADMM Parameters: $\epsilon=0.01$, variable $\gamma = 1$, constant $\lambda = 4$, 2 COFI Iterations, 30 TV iterations, 30 ADMM Iterations

First Results Comparisons: Hawaii, Sentinel-1

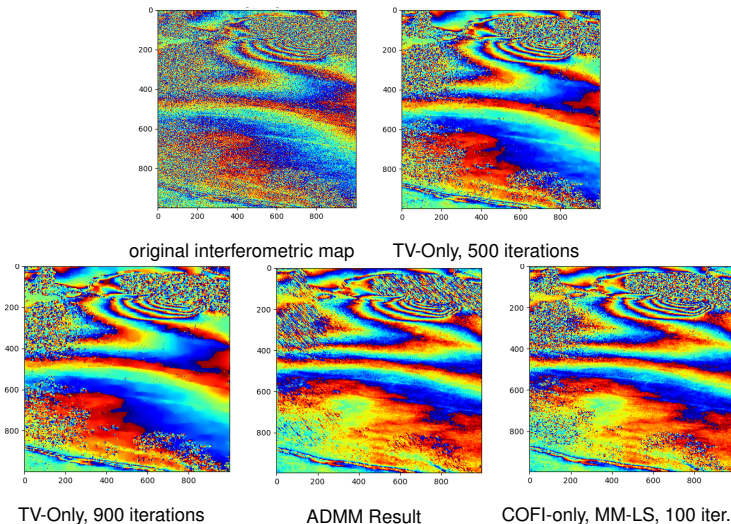


Figure: ADMM Parameters: $\epsilon=0.01$, variable $\gamma = 1$, constant $\lambda = 4$, 2 COFI Iterations, 30 TV iterations, 30 ADMM Iterations

Iterations Analysis

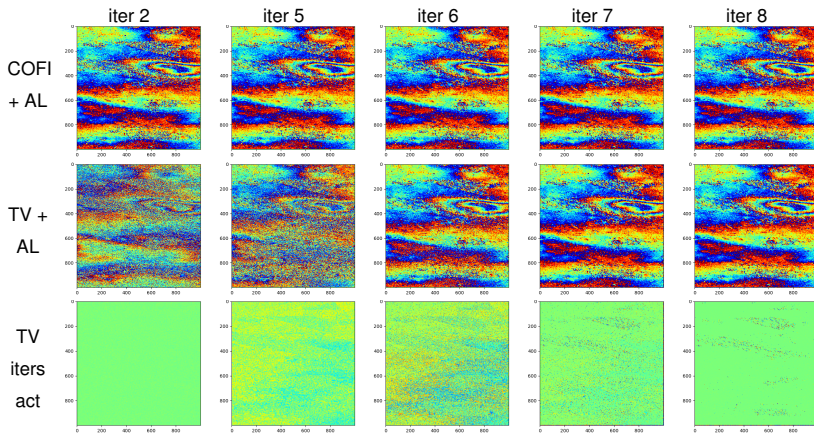
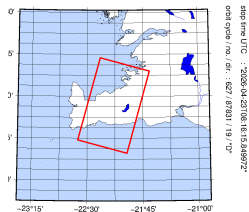
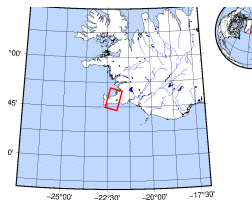
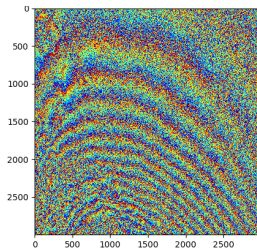


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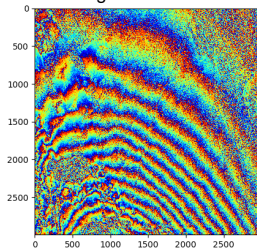
TerraSARX: Iceland



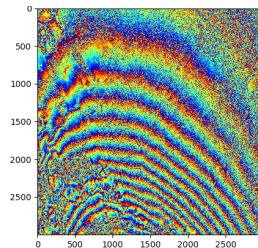
Area of study



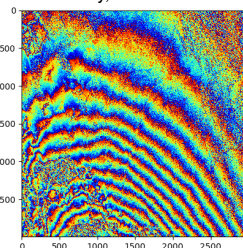
original interf.



COFI-Only



TV-Only, 500 iterations



ADMM

Iterations Analysis

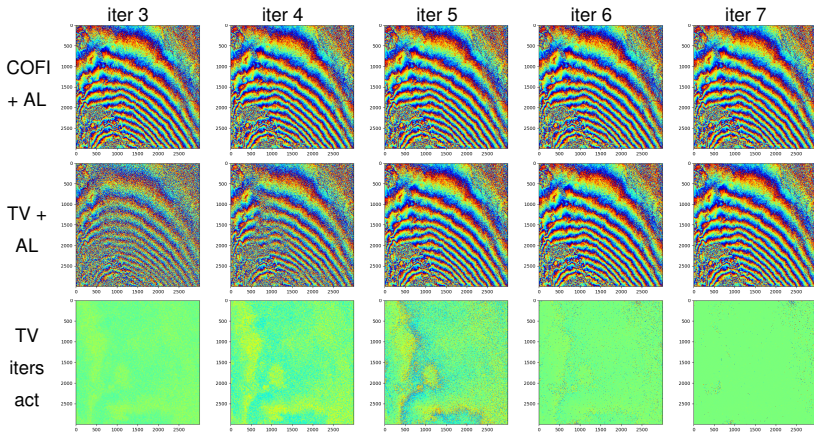


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Conclusions & Perspectives

Conclusions

Adding a spatial regularisation term:

- allows to yield all the **statistical information** of the image stack
- visually eliminates a lot of noise
- Is particularly effective in highly noisy areas that are hard to tackle with the temporal information alone

Perspectives

- Analyse the effects of the various parameters in the framework
- Perform a quantitative evaluation using metrics such as MSE, PSNR, SSIM, and standard deviation reduction.
- Validate the estimated displacements against available GPS ground-truth measurements.

Conclusion

Thank you

References

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- [2] Vu, P. V. H., Breloy, A., Brigui, F., Yan, Y., Ginolhac, G. Robust Phase Linking in In-SAR. IEEE, 2023
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