Adaptive Multi-Scale Estimation of Differential Subsidence from InSAR for Improved Hazard Assessment

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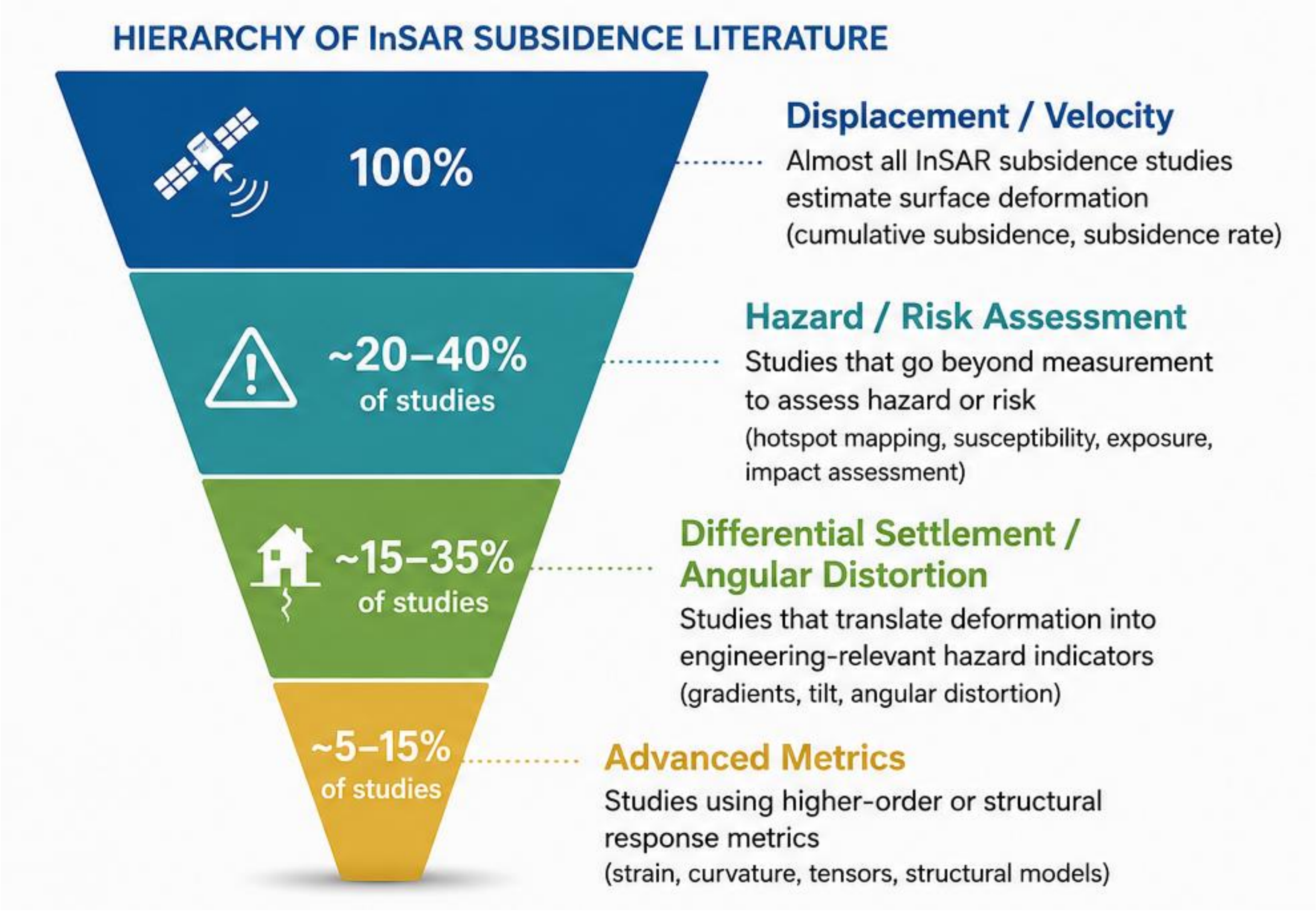
Same AI generated statistics based on a review of recent literature (2010-2024)



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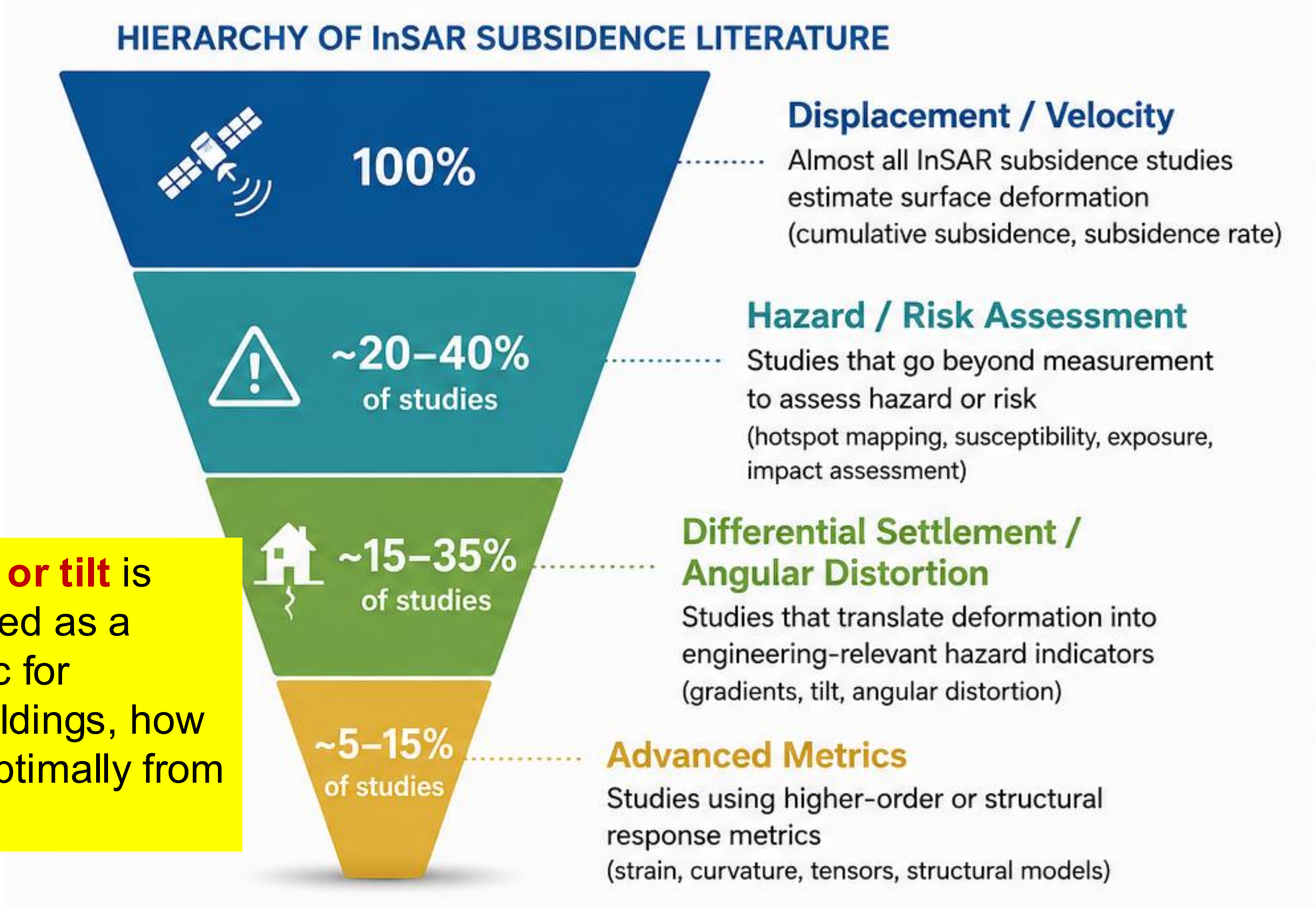


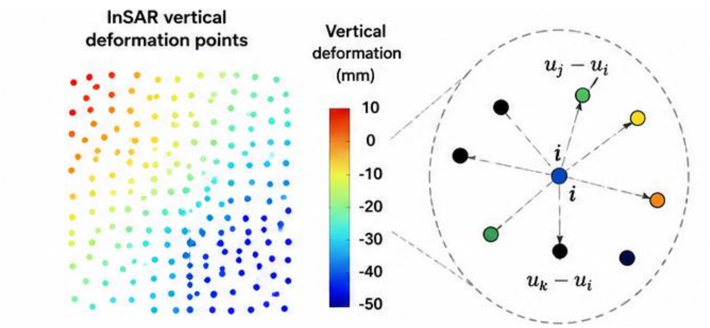
If **angular distortion or tilt** is increasingly recognized as a primary hazard metric for infrastructure and buildings, how can it be estimated optimally from InSAR observations?

Differential Subsidence Estimation

Two Main Approaches for Estimating Differential Subsidence Hazards

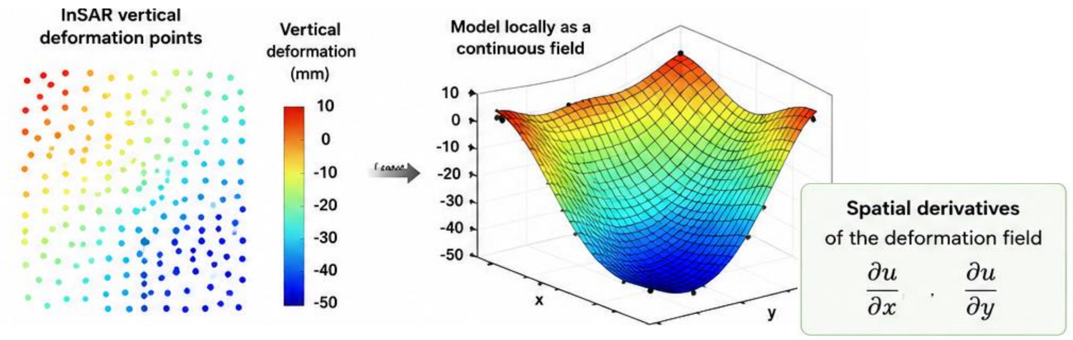
1. Relative Deformation Approach

- Point-to-point displacement differences (divided by distance between points)
- Direct estimation of differential settlement



2. Gradient-Based Approach

- Derivatives of the deformation field
- Estimation of tilt, angular distortion (or curvature)
- Requires **local** modelling of a continuous deformation field



Motivation: Strain/Derivative estimation from geodetic data

Background

• Estimation of strain/gradients from deformation observations requires spatial interpolation of displacement data.

• Interpolation is performed locally around each evaluation point.

Two key interpolation choices:

1. **Basis function:** Most studies assume a *linear plane* (constant strain within window).
2. **Window size:** Defines the spatial scale over which strain parameters are estimated.

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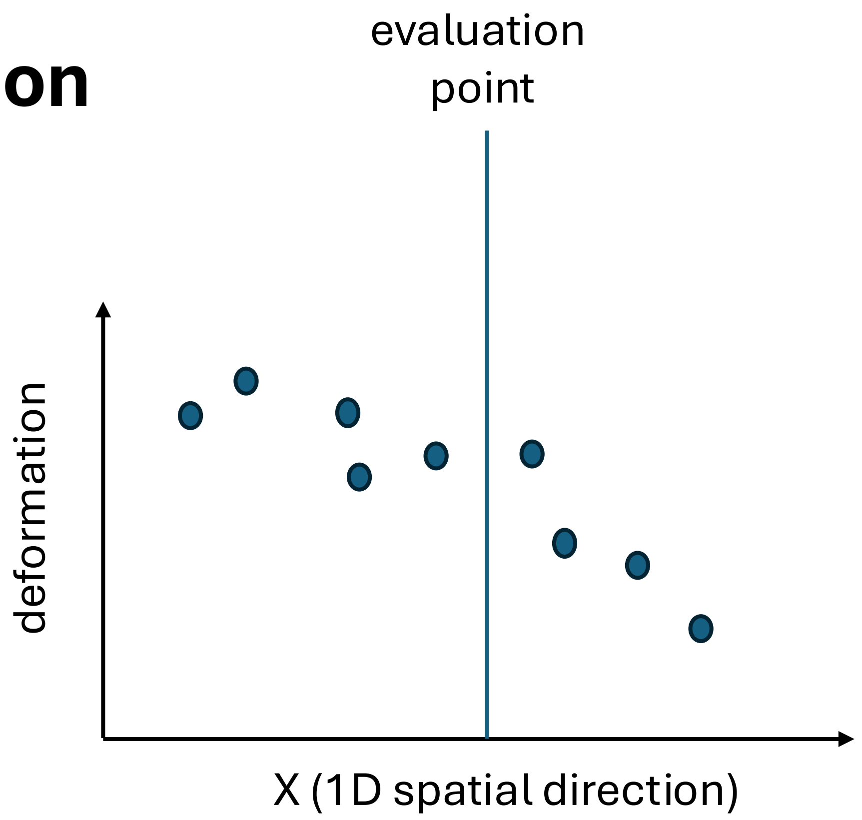

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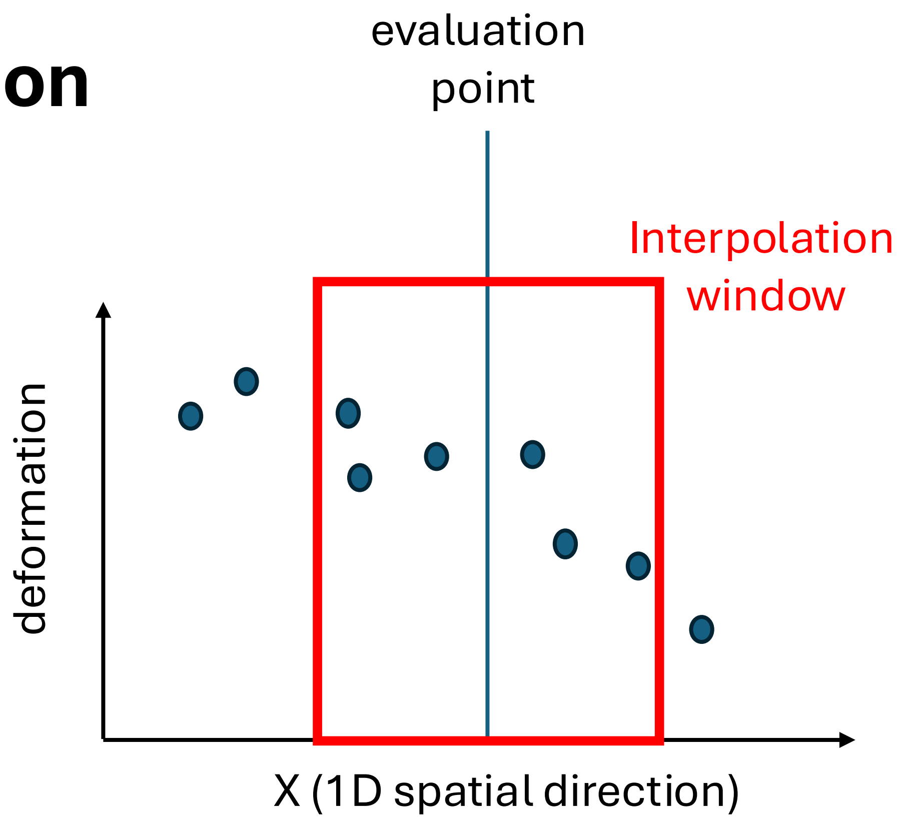

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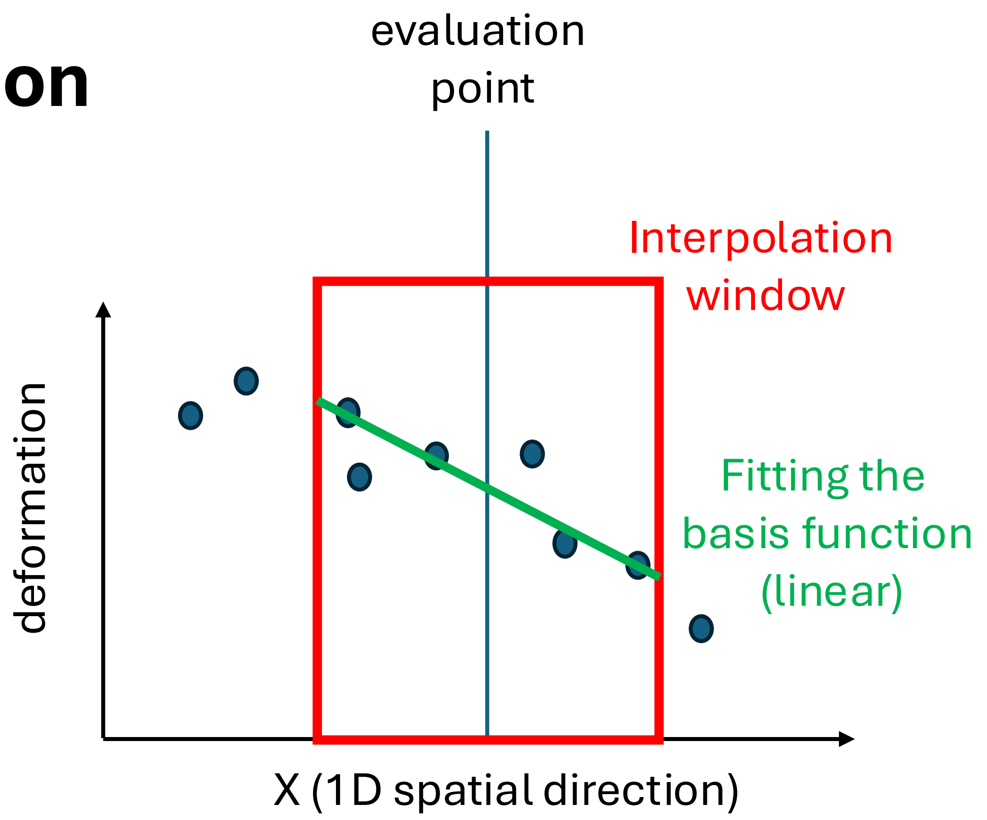

$$\boxed{d = a_0 + a_1 x}$$

Gradient in X direction $\frac{\partial(d)}{\partial(x)} = a_1$

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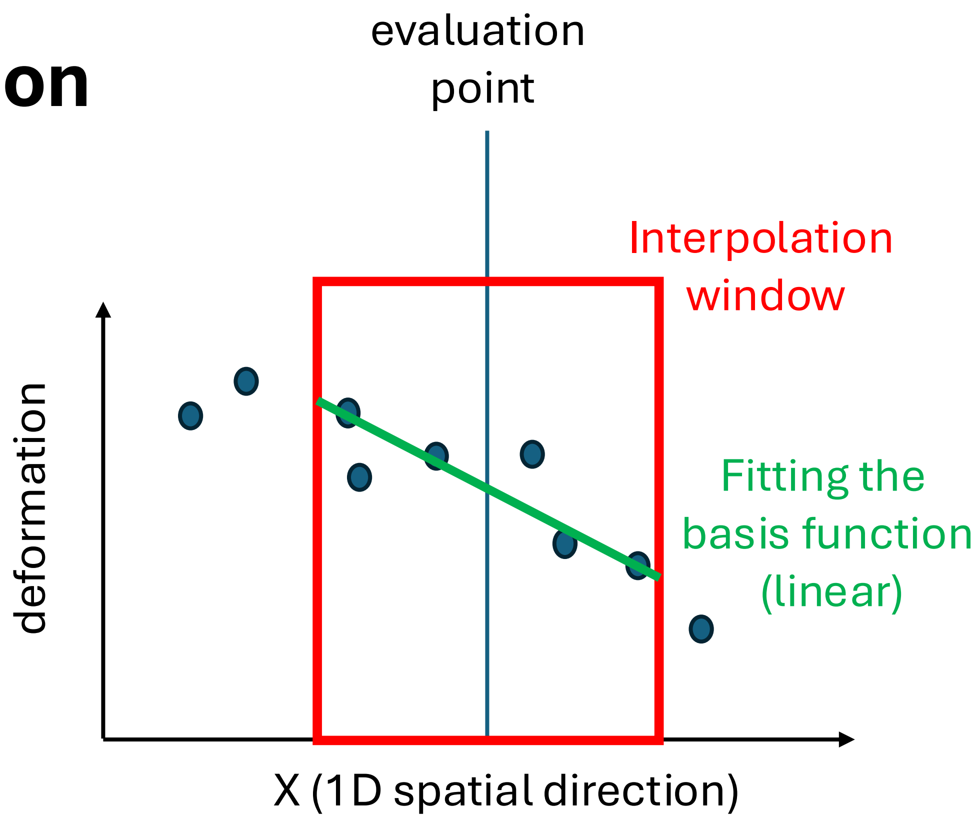

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In 2D case: leniar plane $d = a_0 + a_1 x + a_2 y$

Gradient in X and Y direction $\dfrac{\partial(d)}{\partial(x)} = a_1 \quad \dfrac{\partial(d)}{\partial(y)} = a_2$

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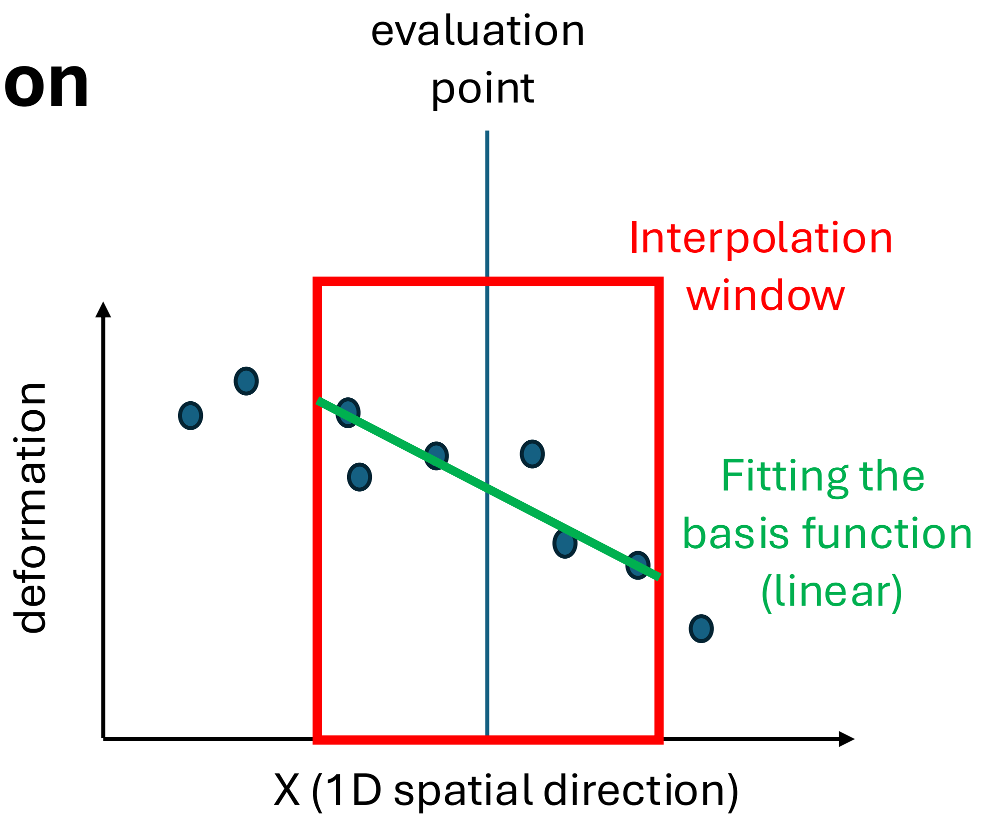
Key requirement for accurate estimation

The true deformation field must be **well represented by the chosen basis function within the interpolation window**.



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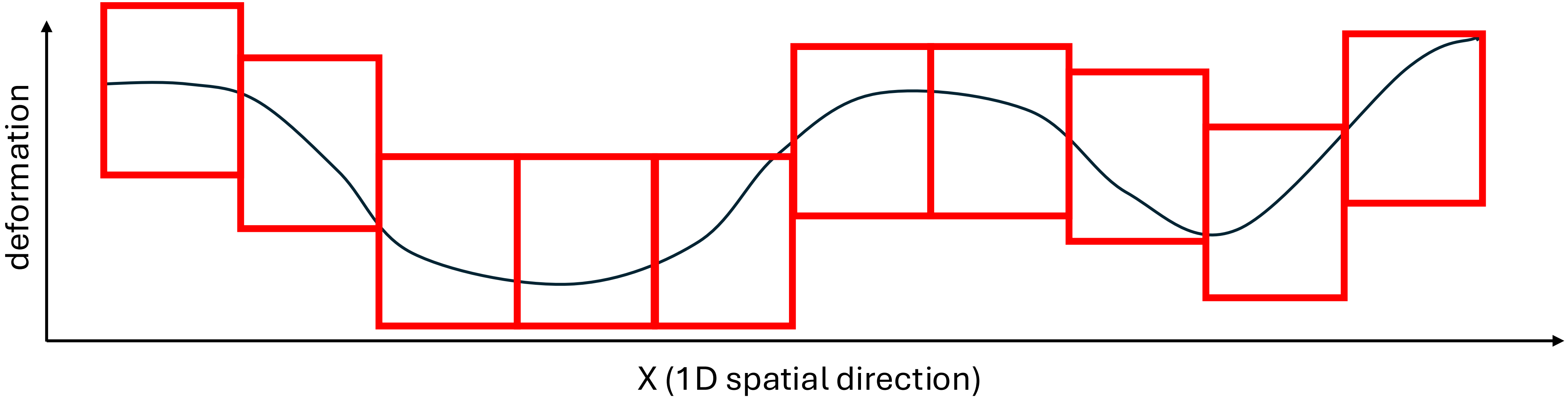
When the assumption is valid

in areas with smooth deformation signal compared to the window size



When the assumption fails

in areas with:

- High spatial-frequency deformation
- Strong gradients
- Localized deformation signals

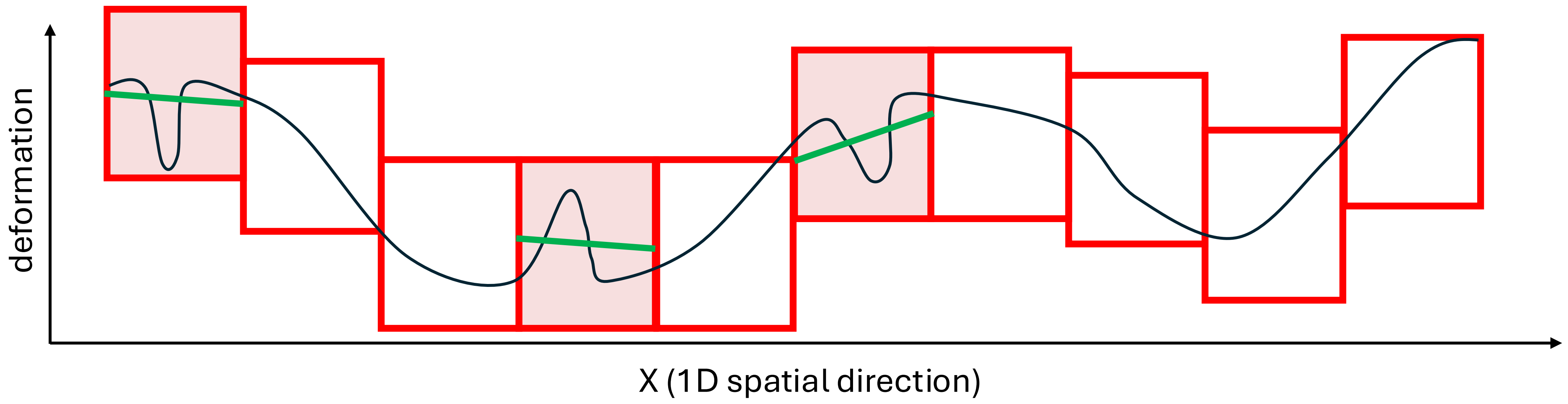
Consequences

Linear basis + fixed window size:

- Cannot represent complex deformation
- Leads to **systematic underestimation of strain / gradients**

Where this commonly occurs

- **Subsidence studies**
    - Localized building-scale subsidence
    - Superimposed on regional subsidence (e.g., groundwater withdrawal)
- **Fault zones**
    - High deformation gradients near faults
    - Complex fault systems with varying geometry, depth, and scale



Proposed approach: flexible basis + adaptive windows

Goal: Improve strain/gradient estimation in complex, multi-scale deformation fields.

Proposed strategy (initial focus: subsidence areas)

Adaptive window size

1. Start with an initial window size
2. **Perform hypothesis testing on basis function validity**
3. If rejected → reduce window size and repeat
4. Continue iteratively until model assumptions are satisfied

$$\Delta D_v(x,y) \approx \frac{\partial D_v}{\partial x}\Delta x + \frac{\partial D_v}{\partial y}\Delta y + c,$$

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In 2D we fit a spatial plane to the deformation feild

$$\Delta D_v(x,y) \approx \frac{\partial D_v}{\partial x}\Delta x + \frac{\partial D_v}{\partial y}\Delta y + c,$$

Design matrix A

$$\begin{bmatrix} \Delta D_{v1} \\ \Delta D_{v2} \\ \Delta D_{v3} \\ \vdots \\ \Delta D_{vm} \end{bmatrix} = \begin{bmatrix} \Delta x_1 & \Delta y_1 & 1 \\ \Delta x_2 & \Delta y_2 & 1 \\ \Delta x_3 & \Delta y_3 & 1 \\ \vdots & \vdots & \vdots \\ \Delta x_m & \Delta y_m & 1 \end{bmatrix} \begin{bmatrix} \frac{\partial D_v}{\partial x} \\ \frac{\partial D_v}{\partial y} \\ c \end{bmatrix}$$

Relative deformation between the point of interest nnd the neighboring points in the window

derivatives

Vector of unknown ***x***

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Design matrix A

Relative deformation between the point of interest nnd the neighboring points in the window

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derivatives

Vector of unknown ***x***

$$\hat{\mathrm{x}} = \left(\mathrm{A}^T Q_{\mathrm{y}}^{-1} \mathrm{A}\right)^{-1} \mathrm{A}^T Q_{\mathrm{y}}^{-1} \mathrm{y}$$

$$\hat{e} = y - A\hat{x}$$

Misfit measure (Test statistics) $$T = \hat{e}^T Q_y^{-1} \hat{e} \sim \chi^2(m-n)$$

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derivatives

Vector of unknown ***x***

Qy captures the noise variability (and also the correlated noise)

+ allows the error propagation
+ differentiate between gradient of the noise and of the signal

- demanding: we need information about the noise

$$(\mathrm{A}^T Q_{\mathrm{y}}^{-1} \mathrm{A})^{-1} \mathrm{A}^T Q_{\mathrm{y}}^{-1} \mathrm{y}$$

$$\hat{e} = y - A\hat{x}$$

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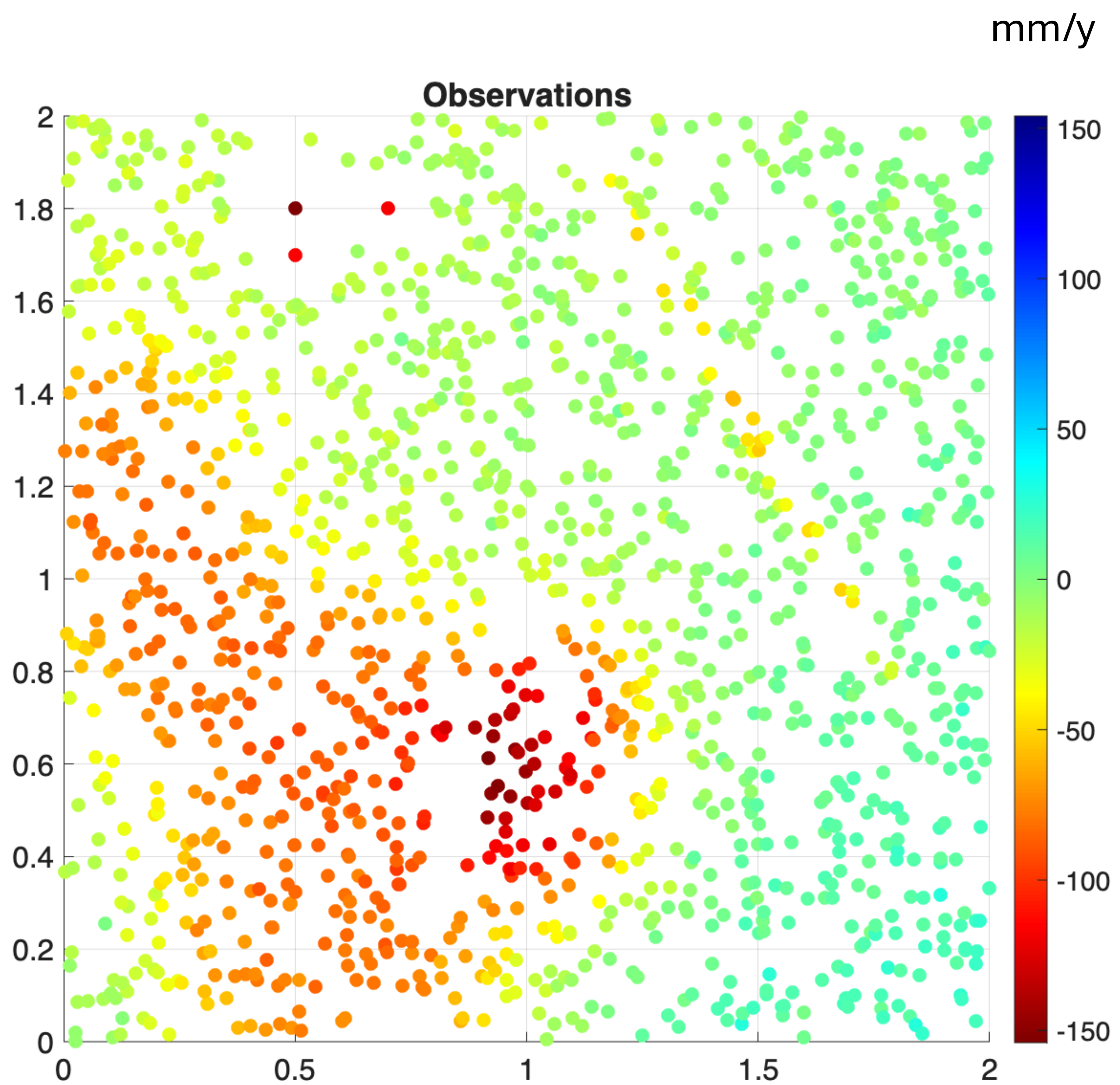
$$T = \hat{e}^T Q_y^{-1} \hat{e} \quad \sim \quad \chi^2(m-n)$$

Let's look at some example (simulated data)

Multi-scale deformation field



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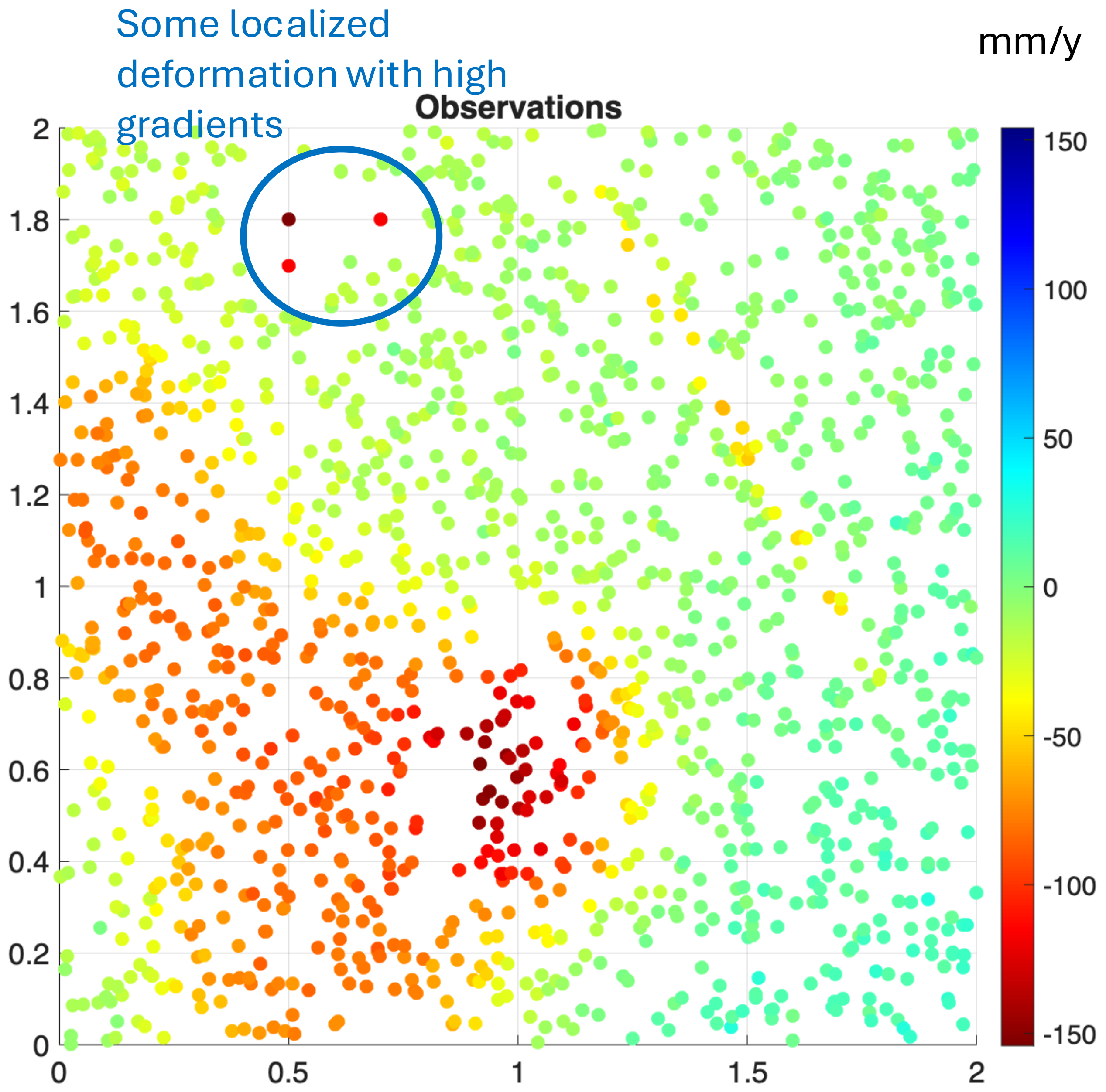

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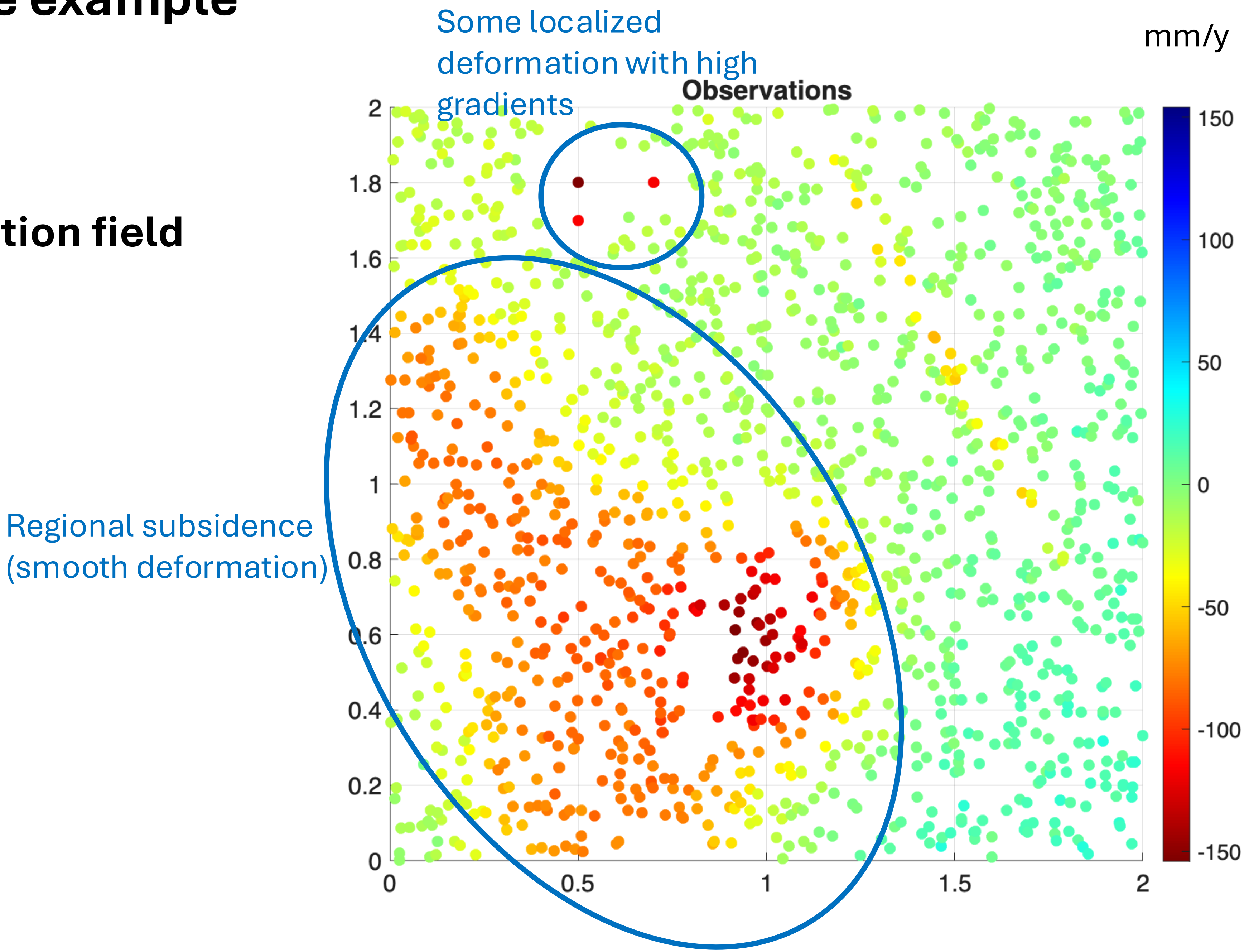

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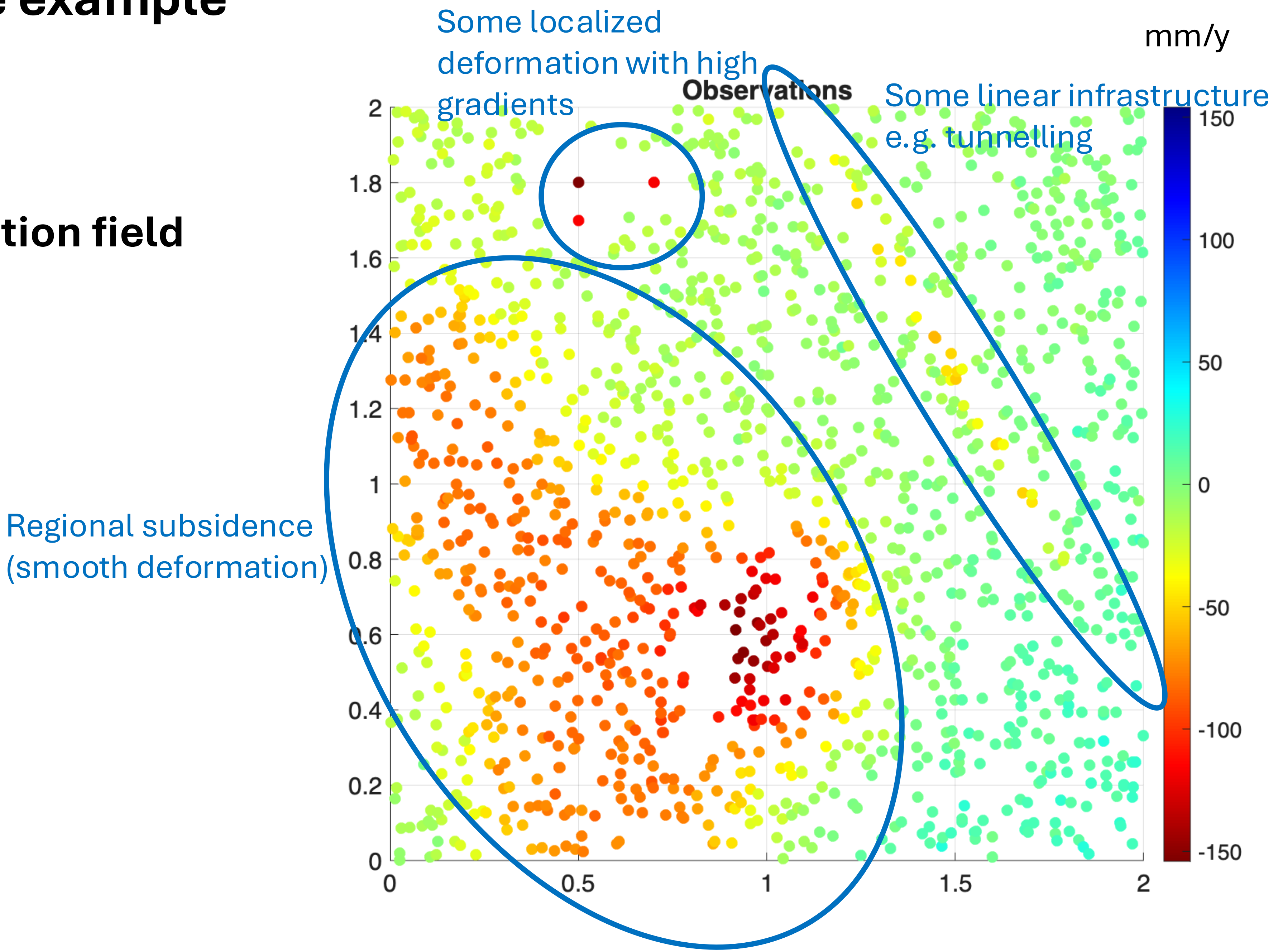

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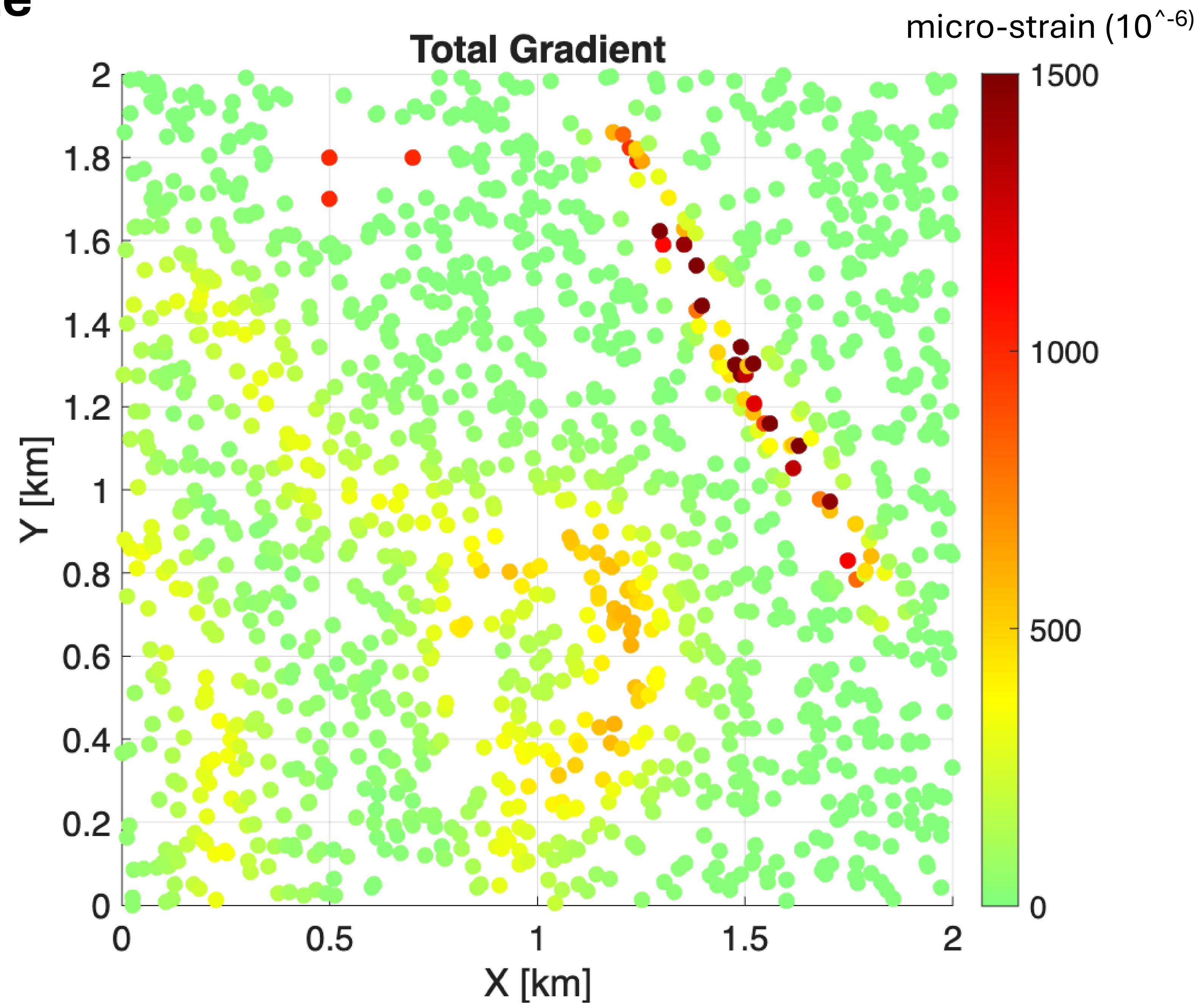
This is the true gradient



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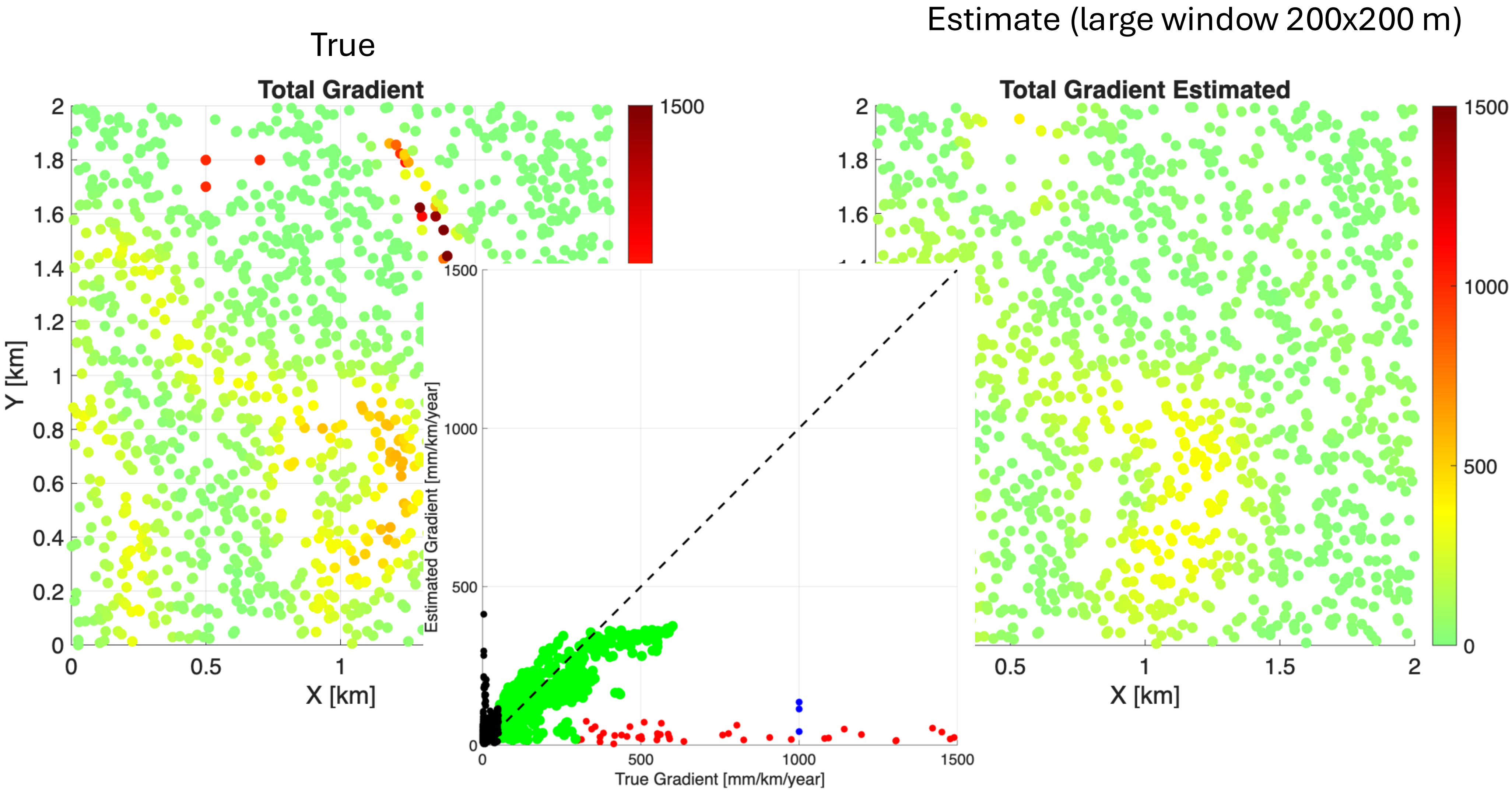

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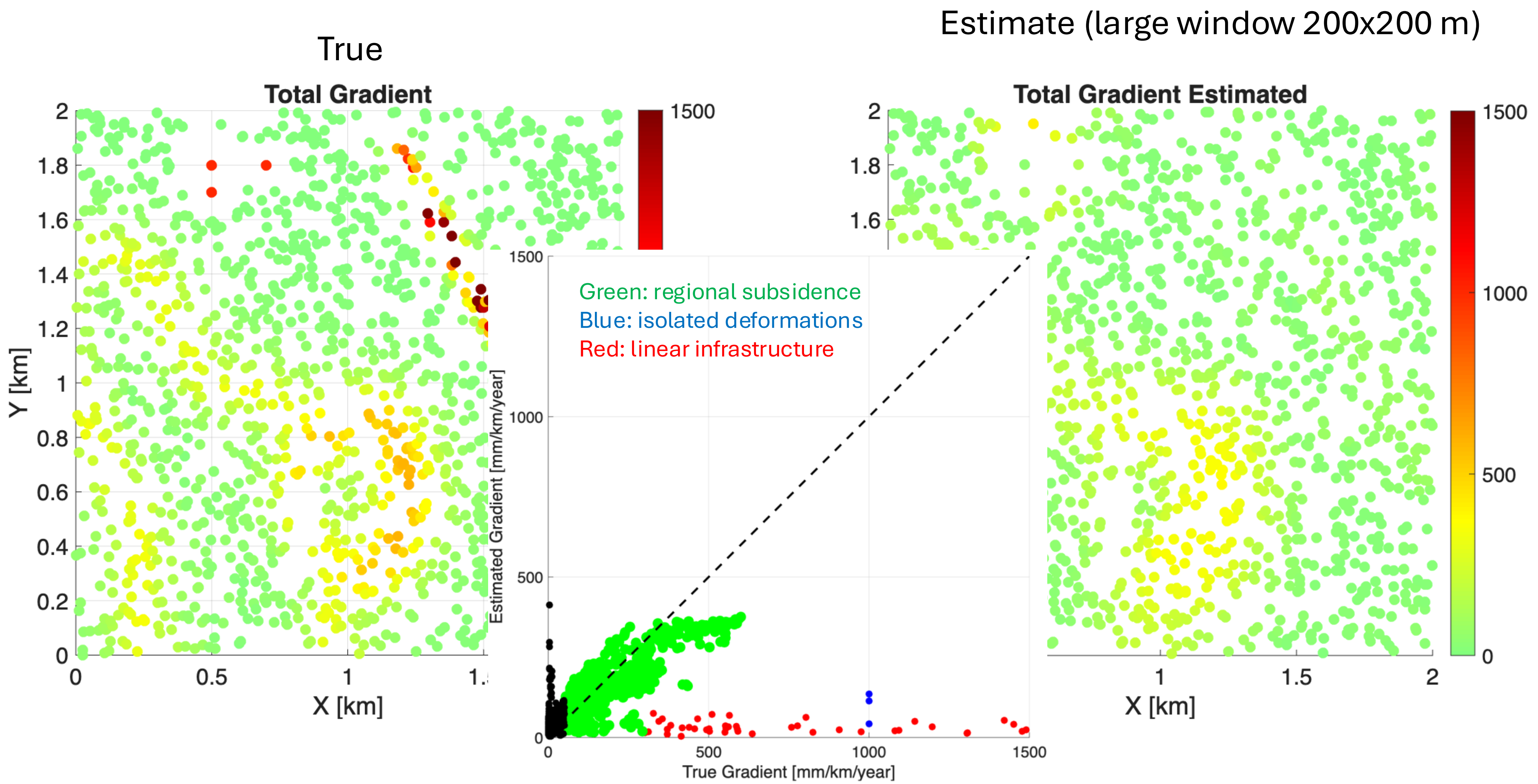

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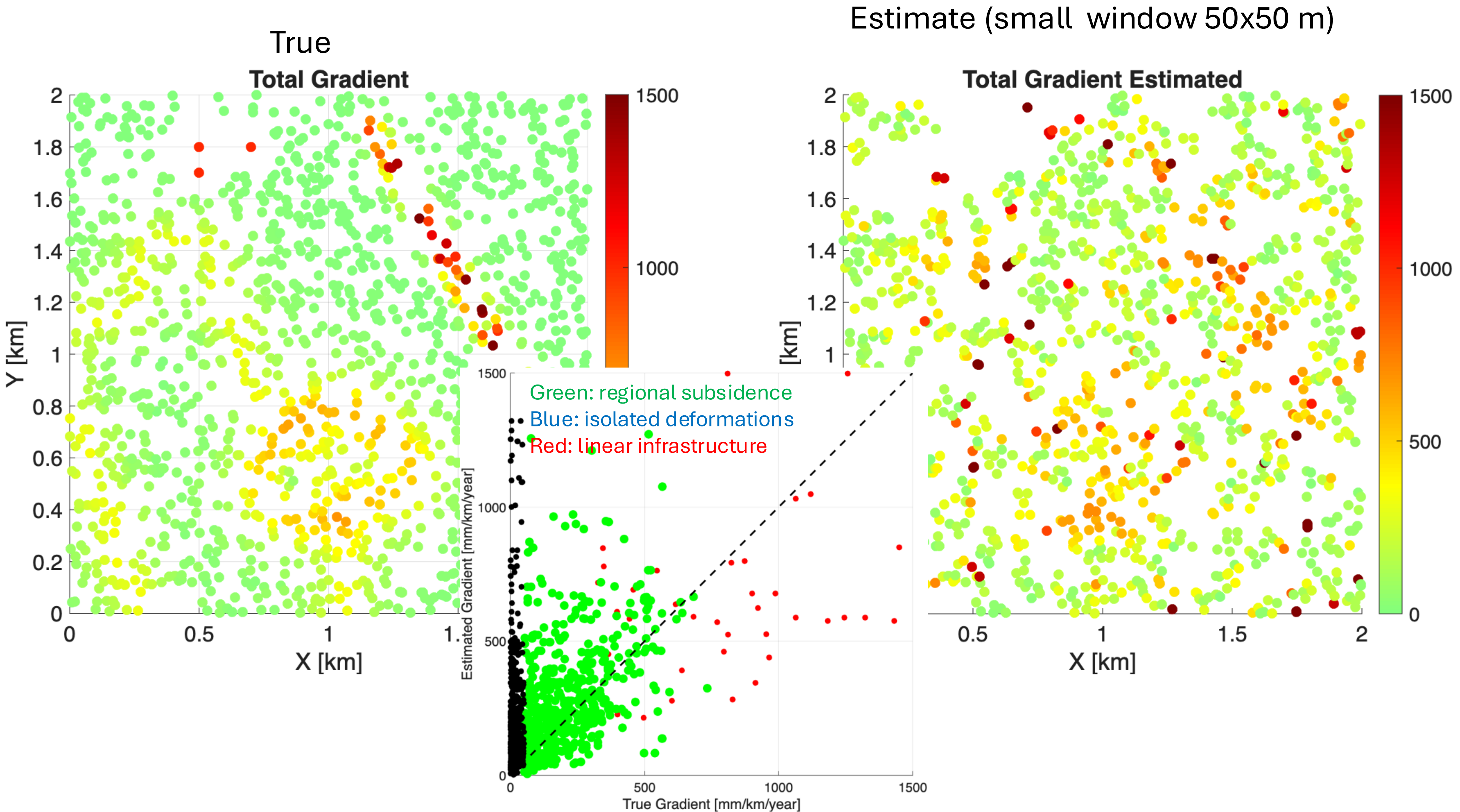

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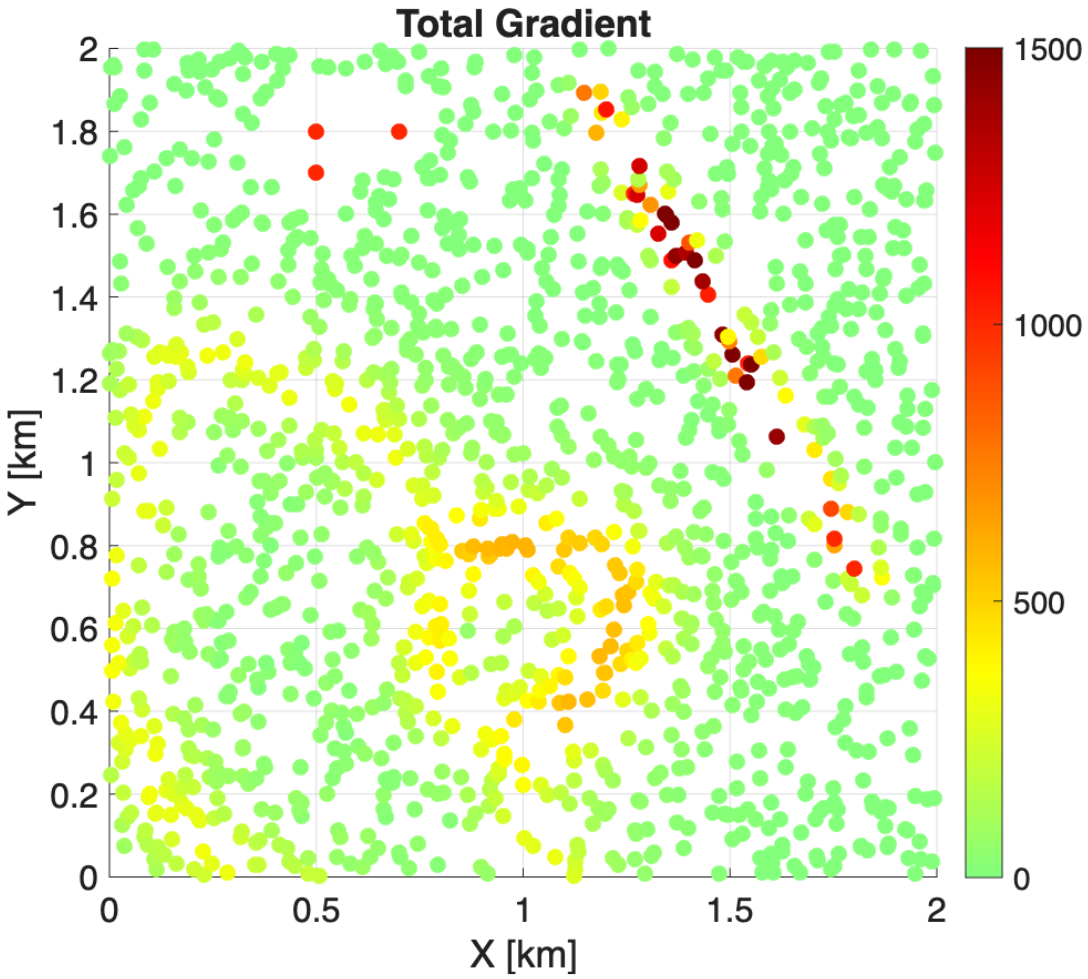
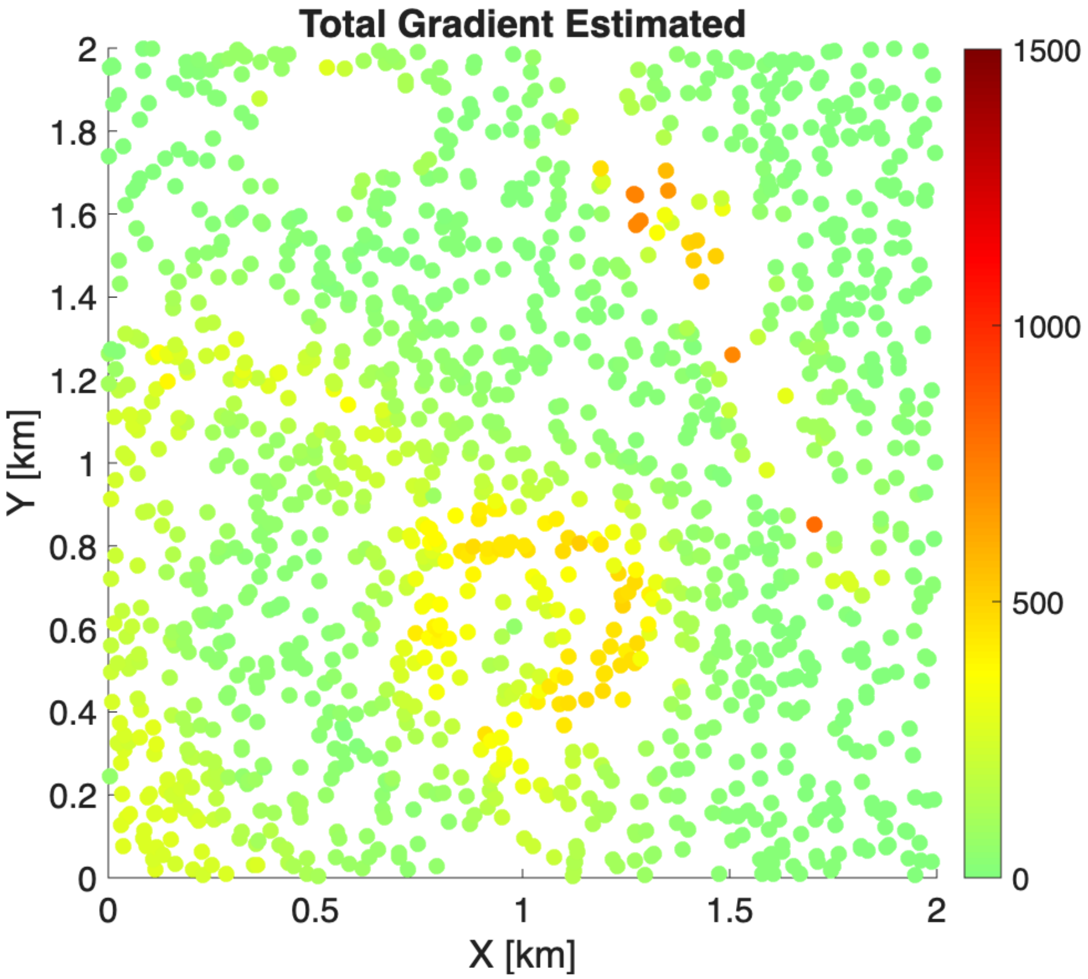
True



Proposed approach (adaptive)



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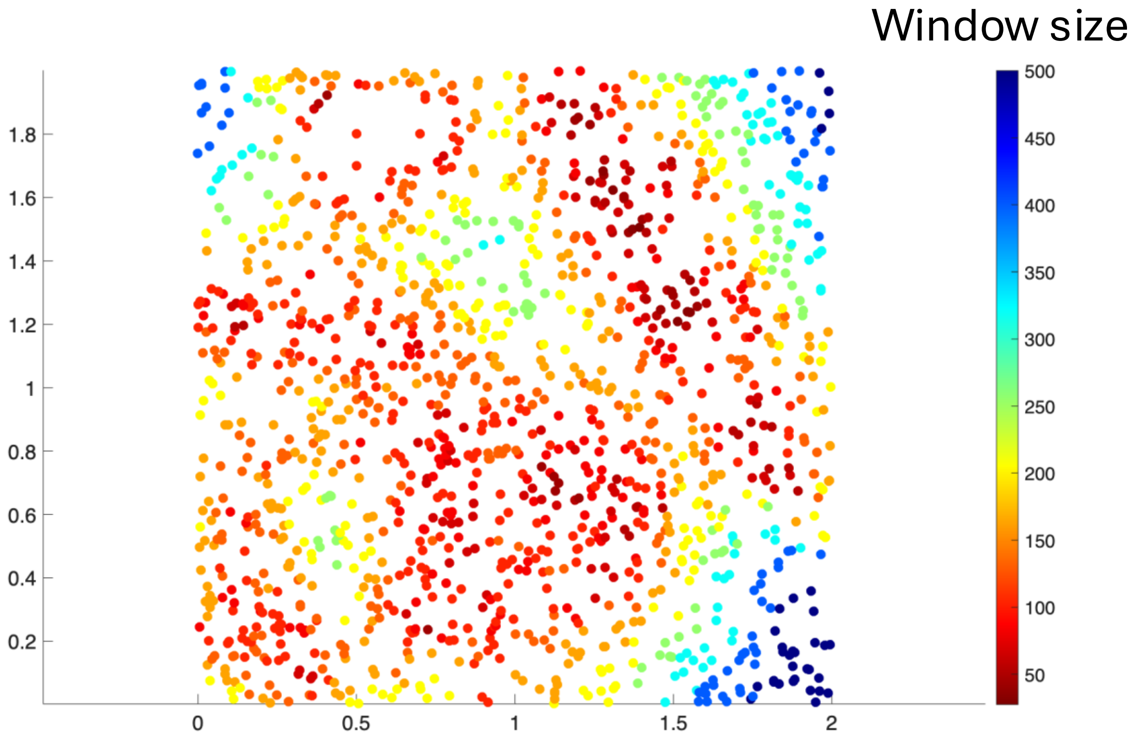
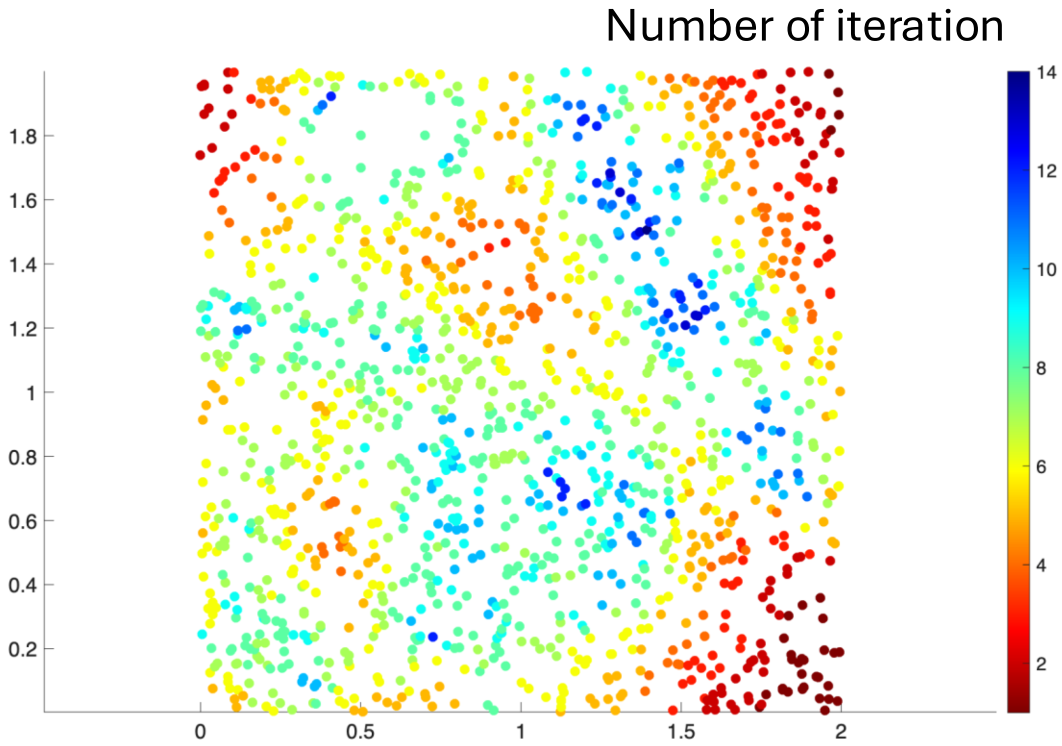
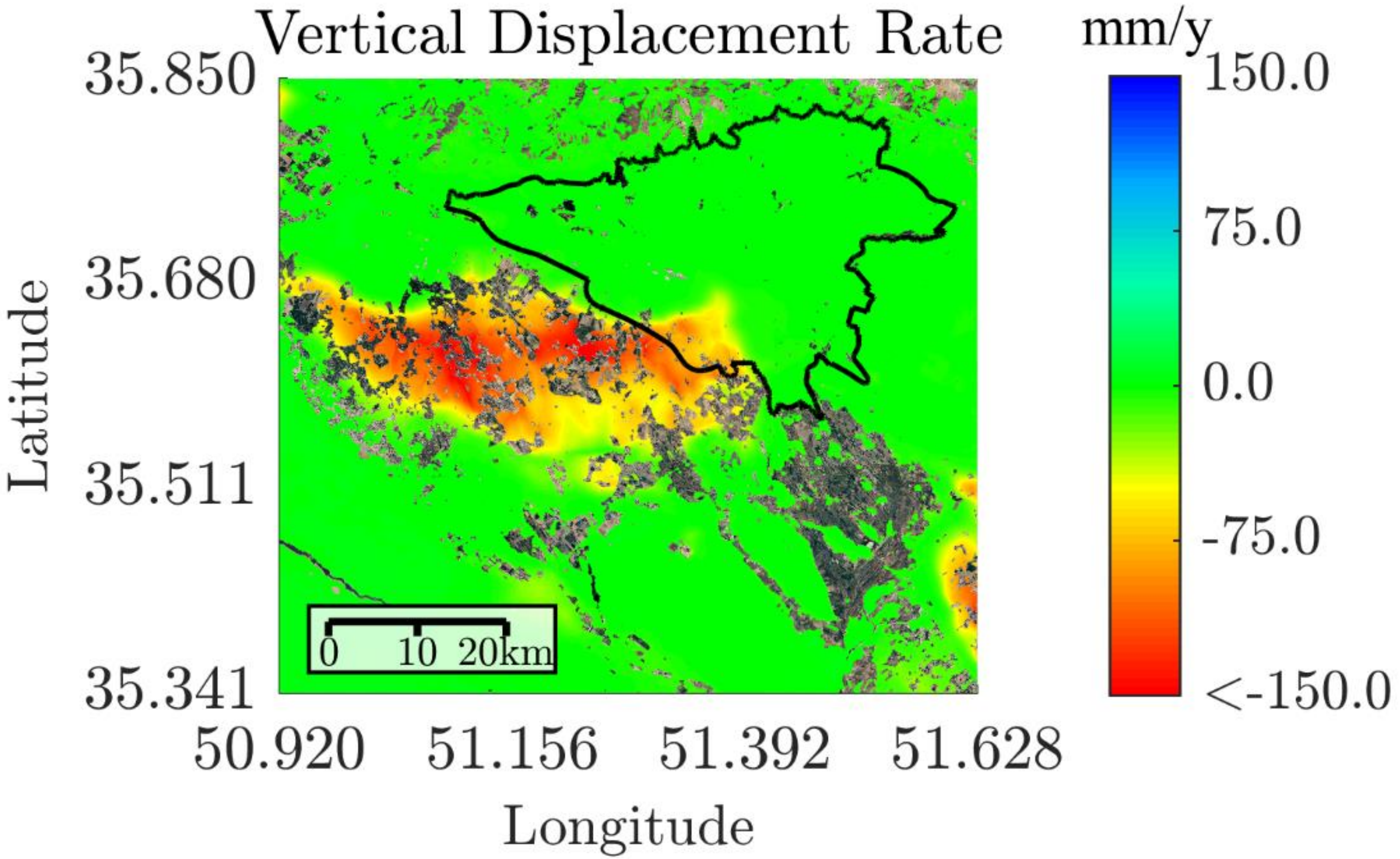
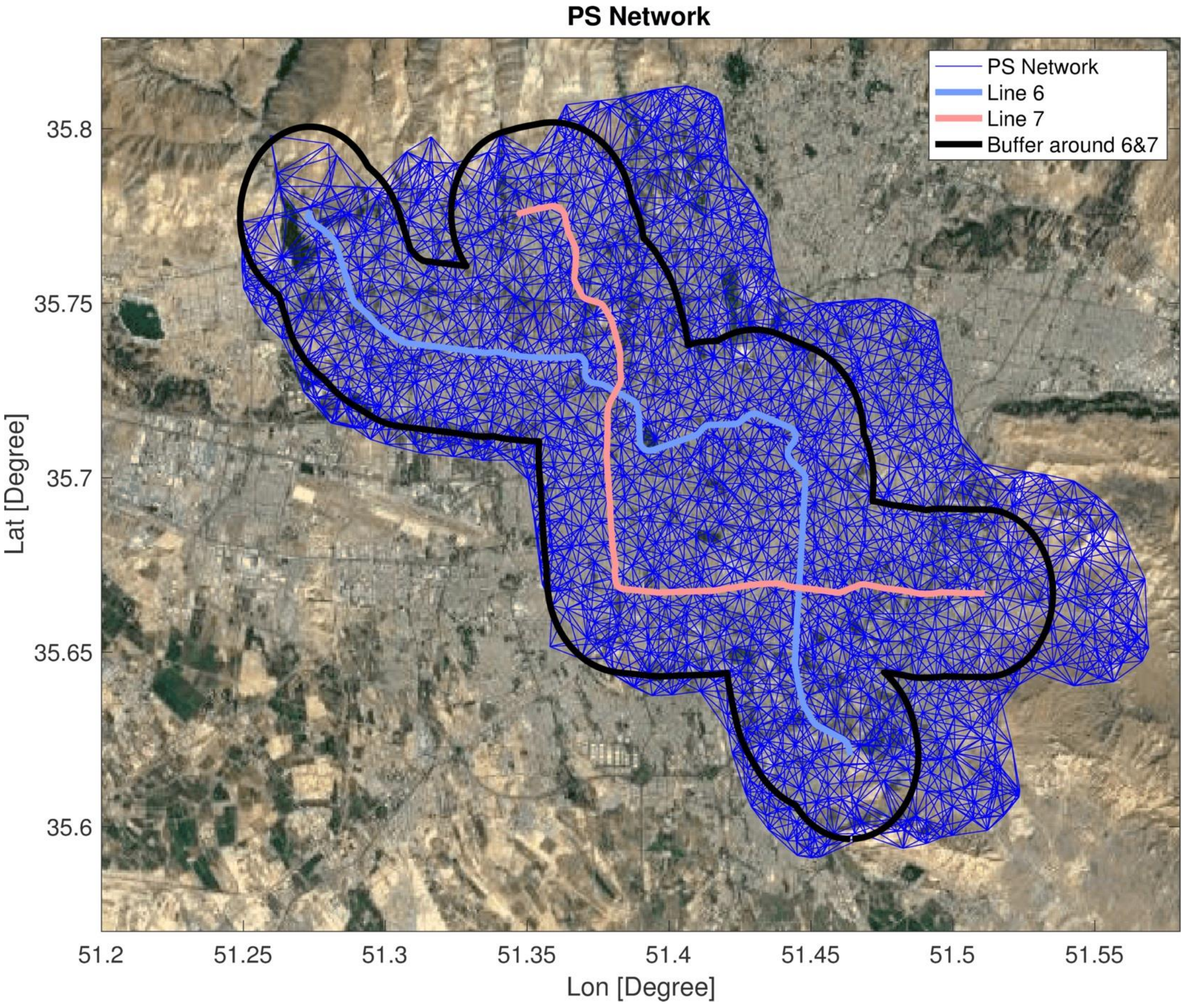




Case study (Tehran)




PS Network
PS Network
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Line 7
Buffer around 6&7
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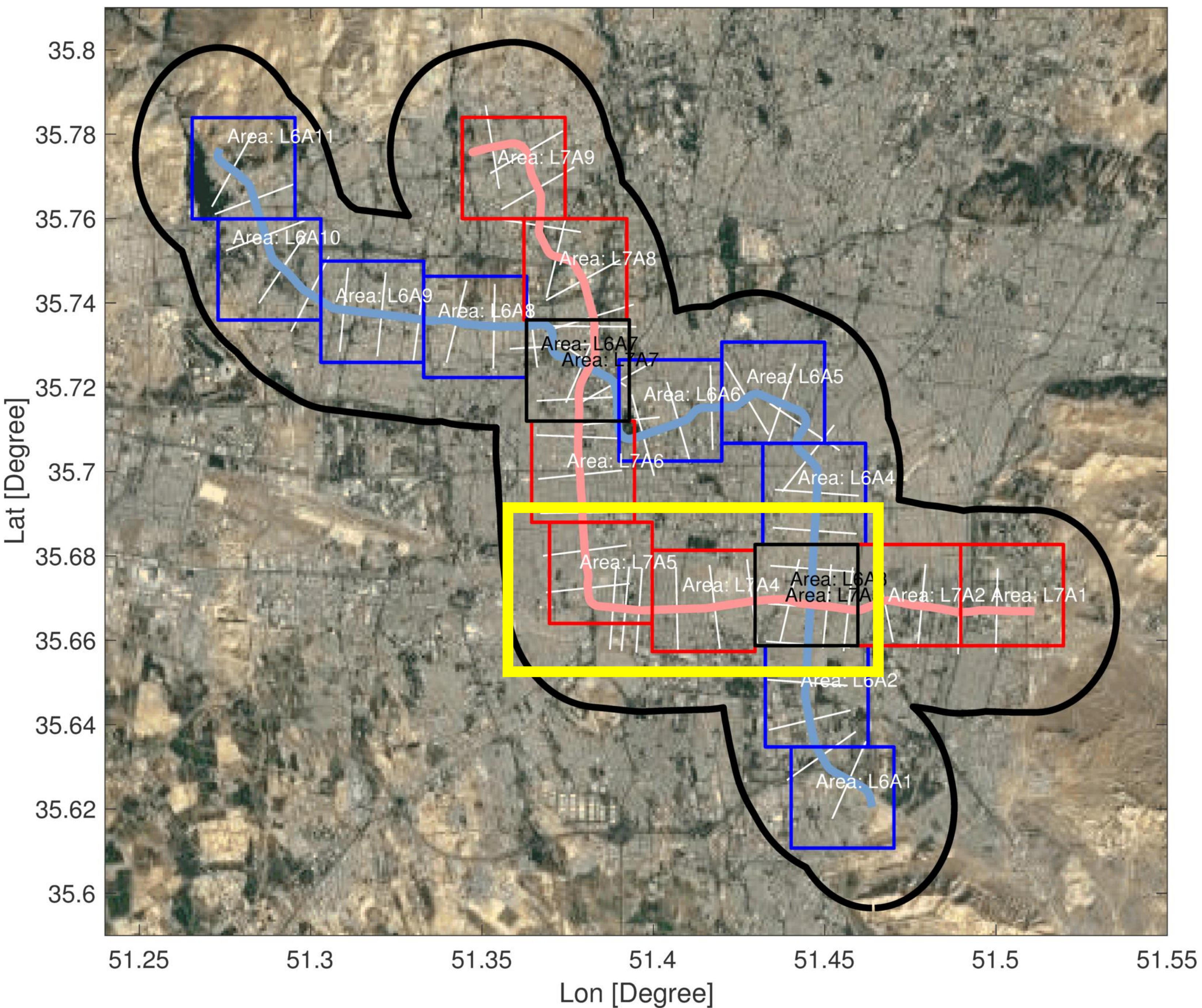

Area: L6A11
Area: L6A10
Area: L6A9
Area: L6A8
Area: L6A7
Area: L7A7
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Area: L6A4
Area: L6A2
Area: L6A1
Area: L7A9
Area: L7A8
Area: L7A6
Area: L7A5
Area: L7A4
Area: L7A2
Area: L7A1
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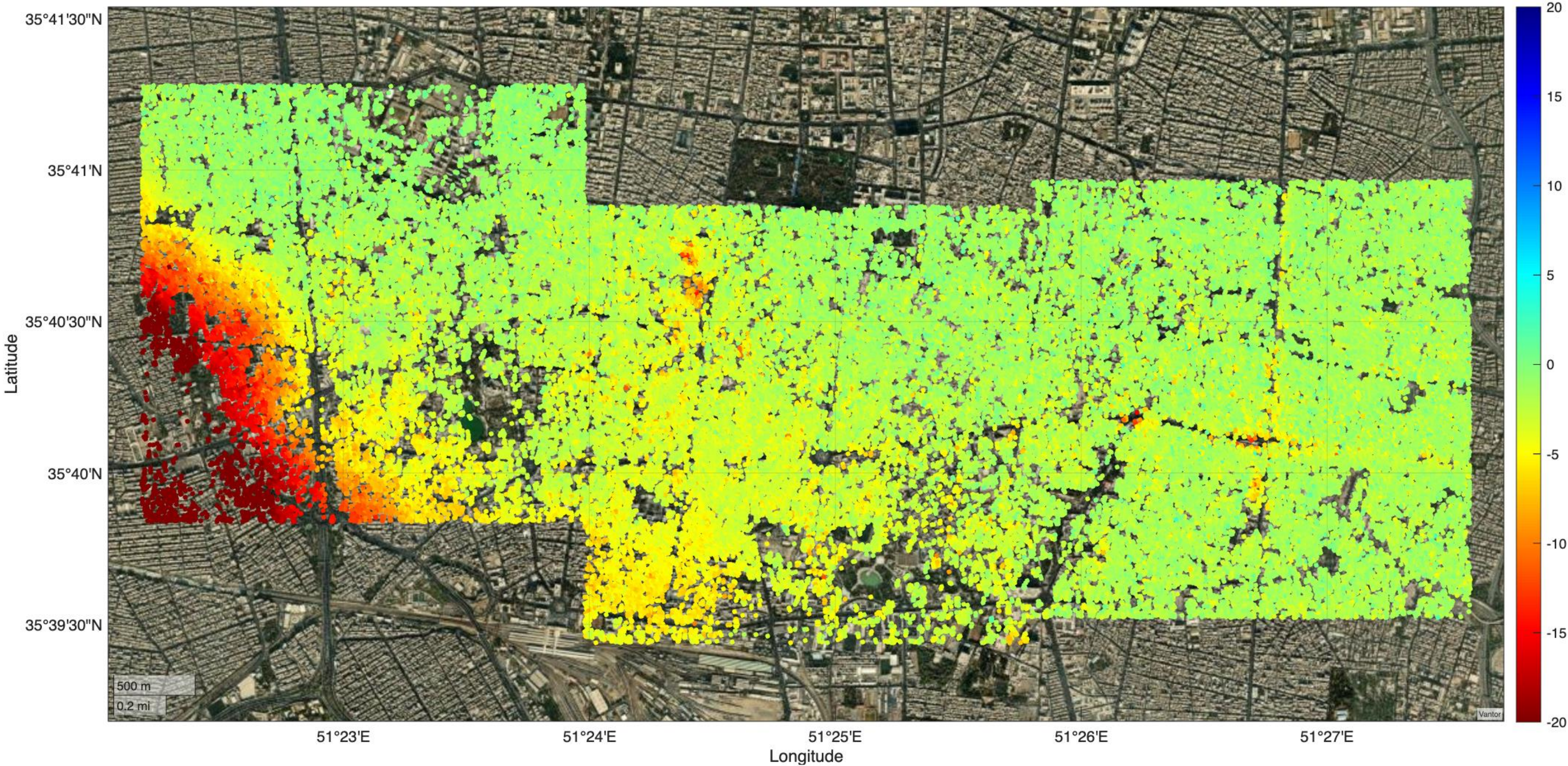
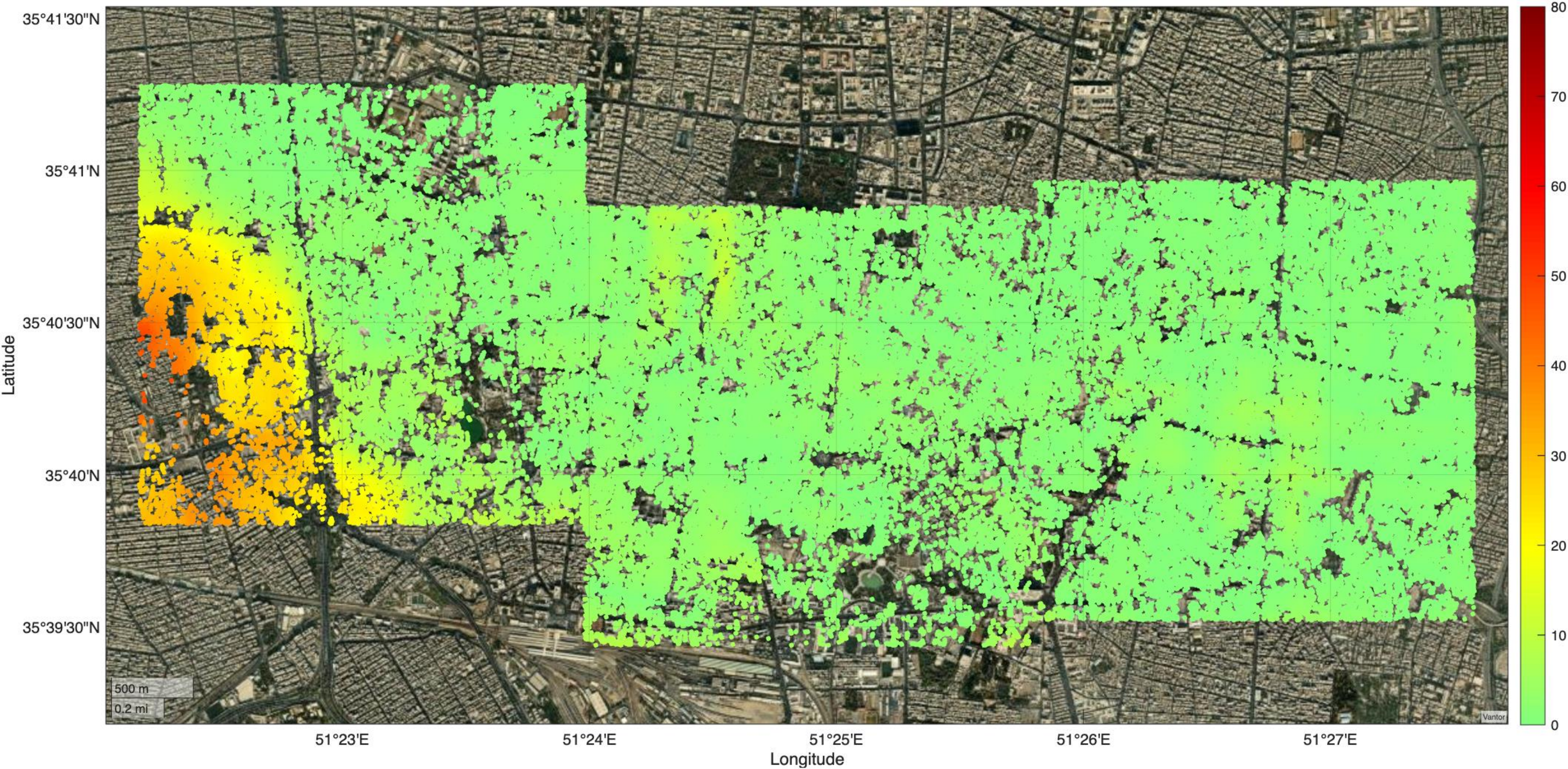
Case study (Tehran): PS velocity map (mm/y)



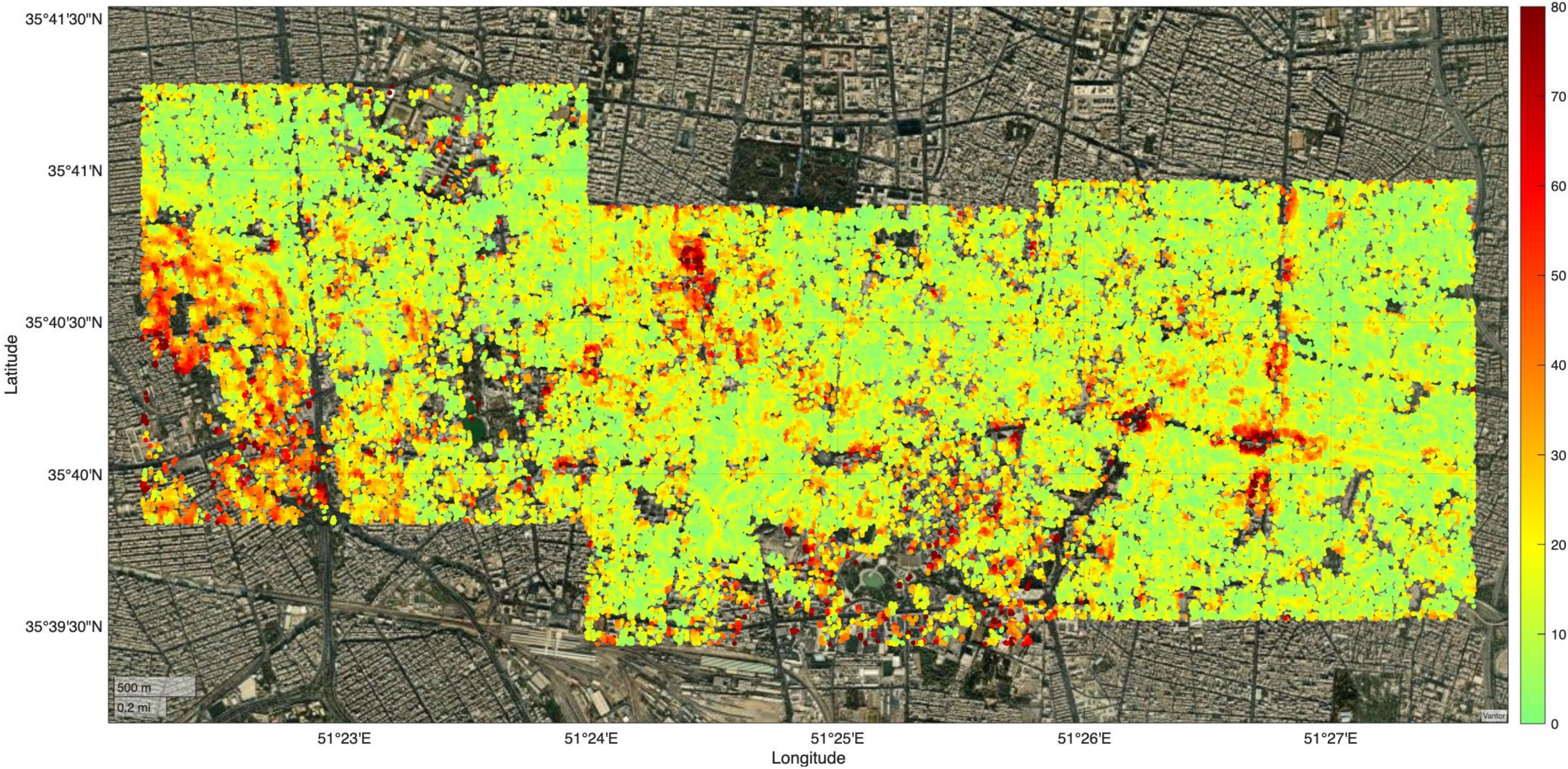
Case study (Tehran): Total gradient [micro strain 10^-6]

Windows size: 200 m



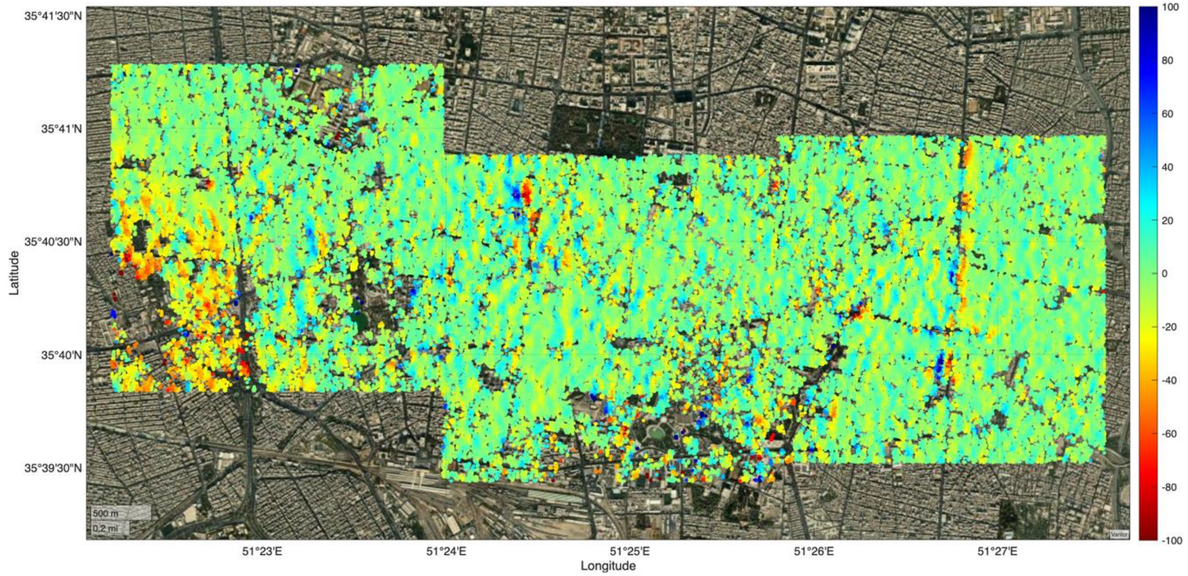
Case study (Tehran): Total gradient [micro strain 10^-6]

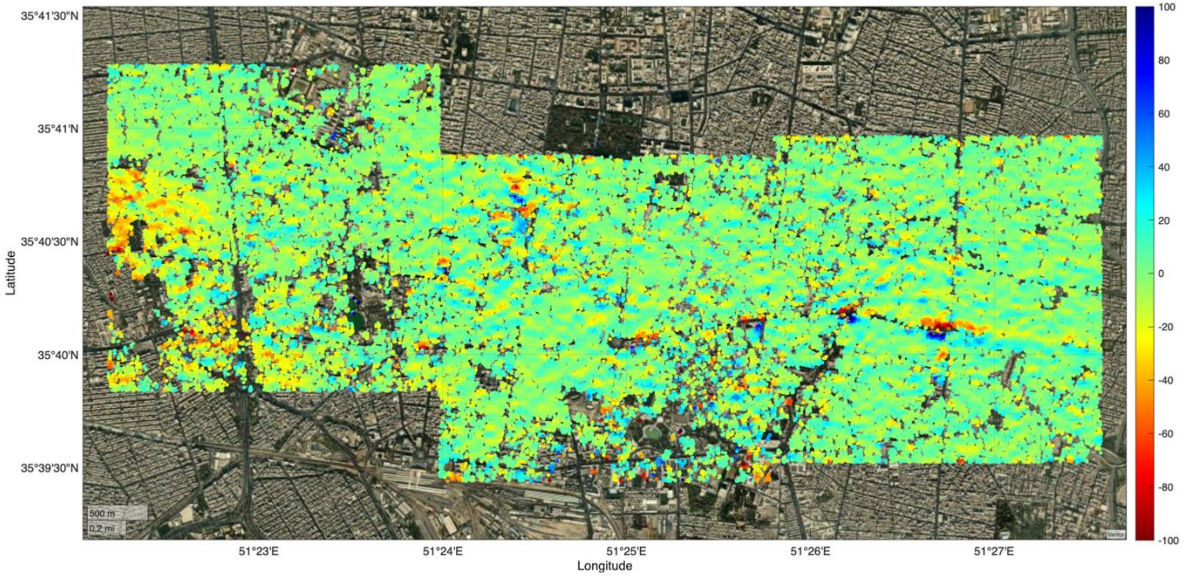
Windows size: 50 m



Case study (Tehran):
North and East components of the tilt [micro strain 10^{-6}]

Windows size: 50 m





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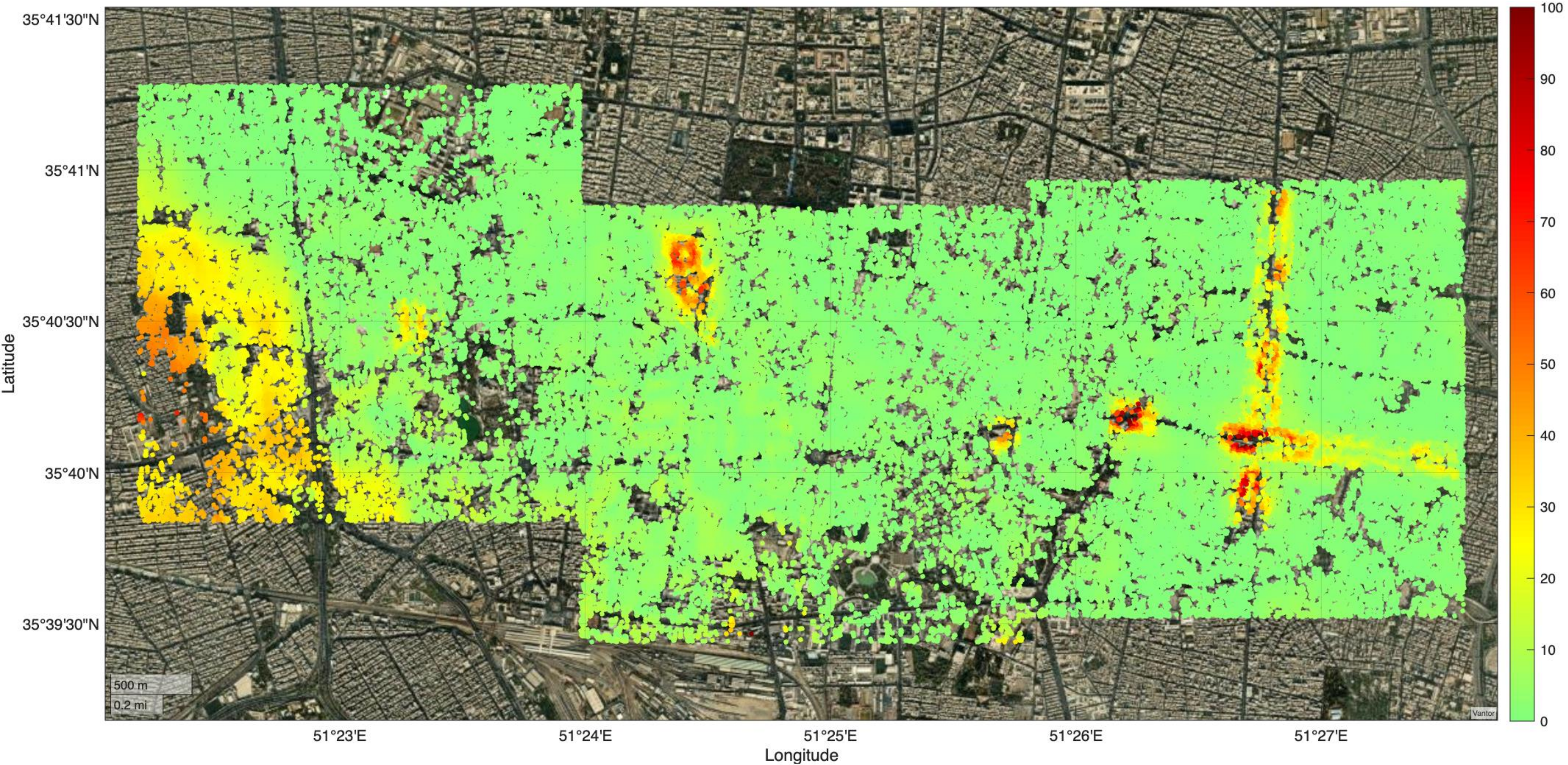
Adaptive method



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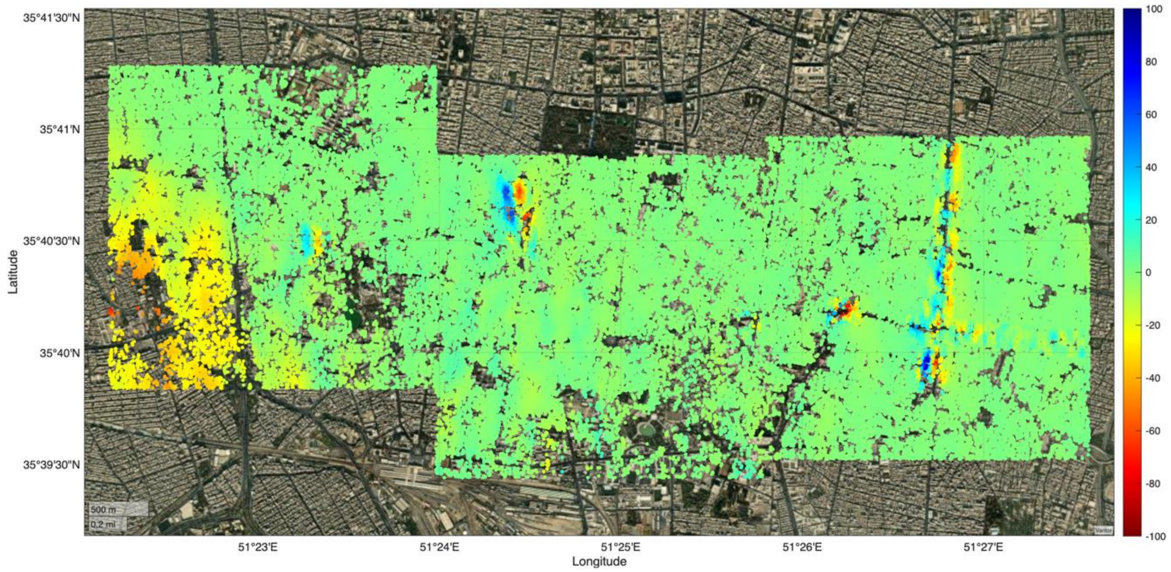
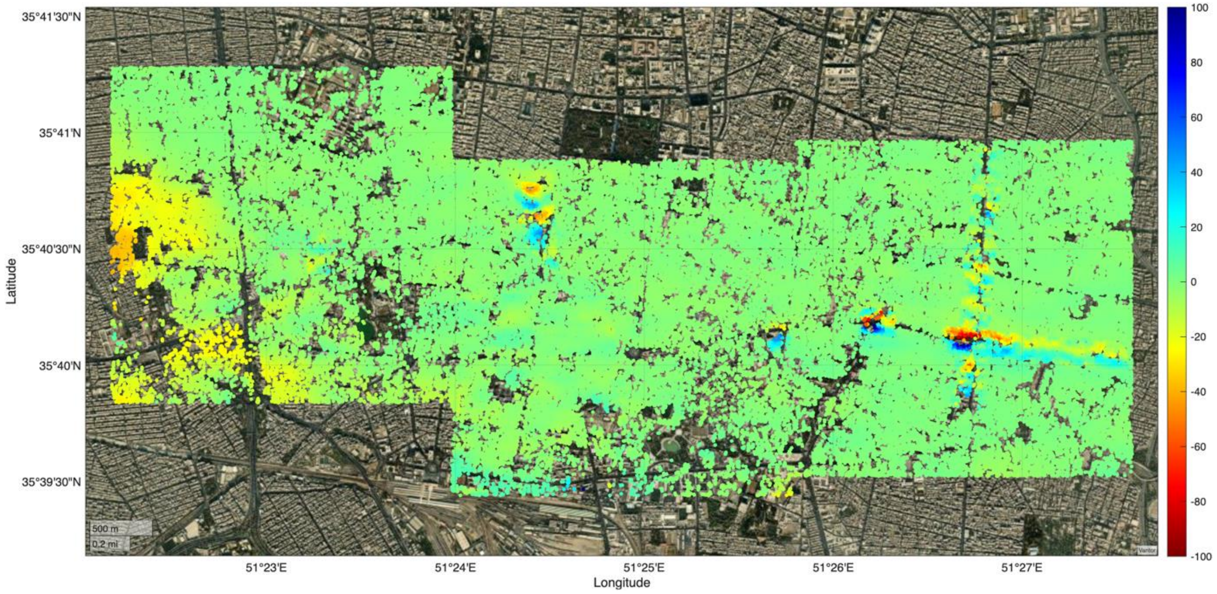
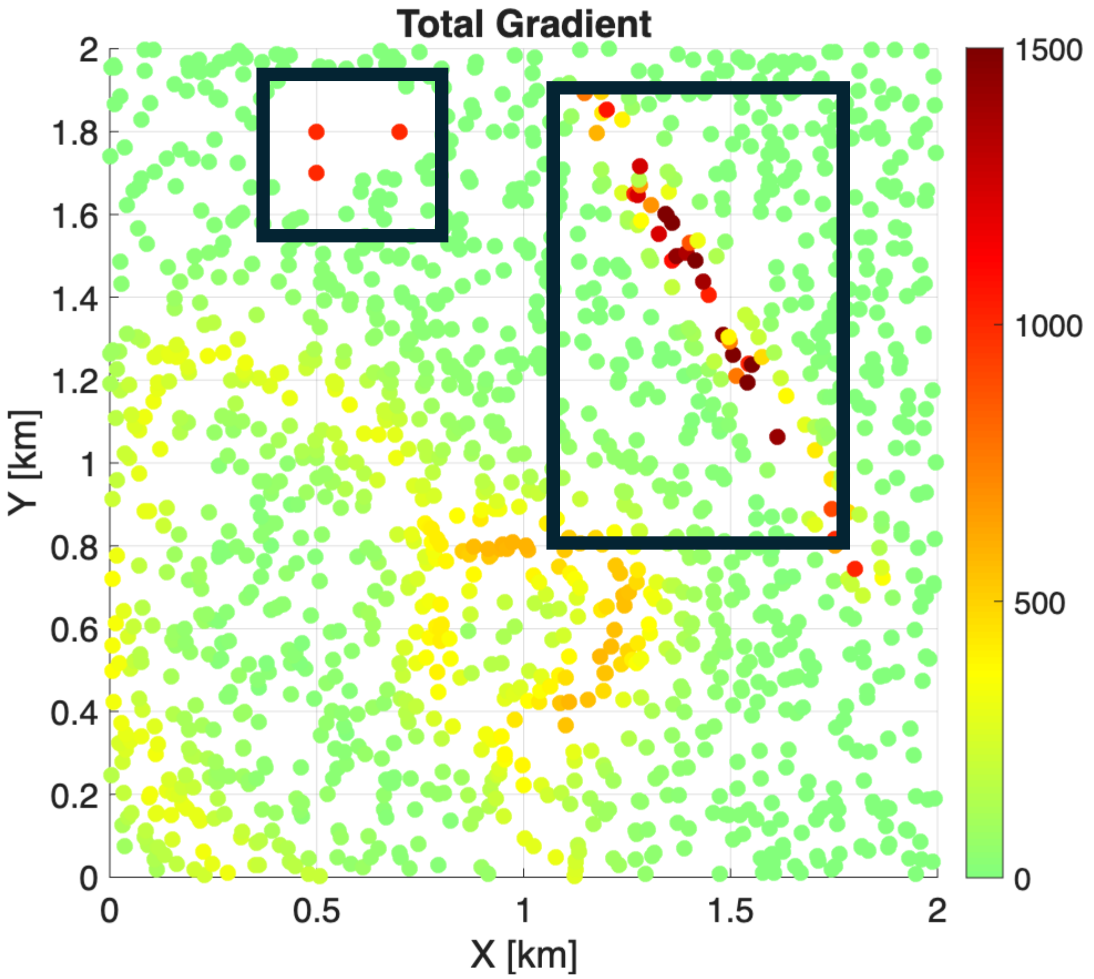


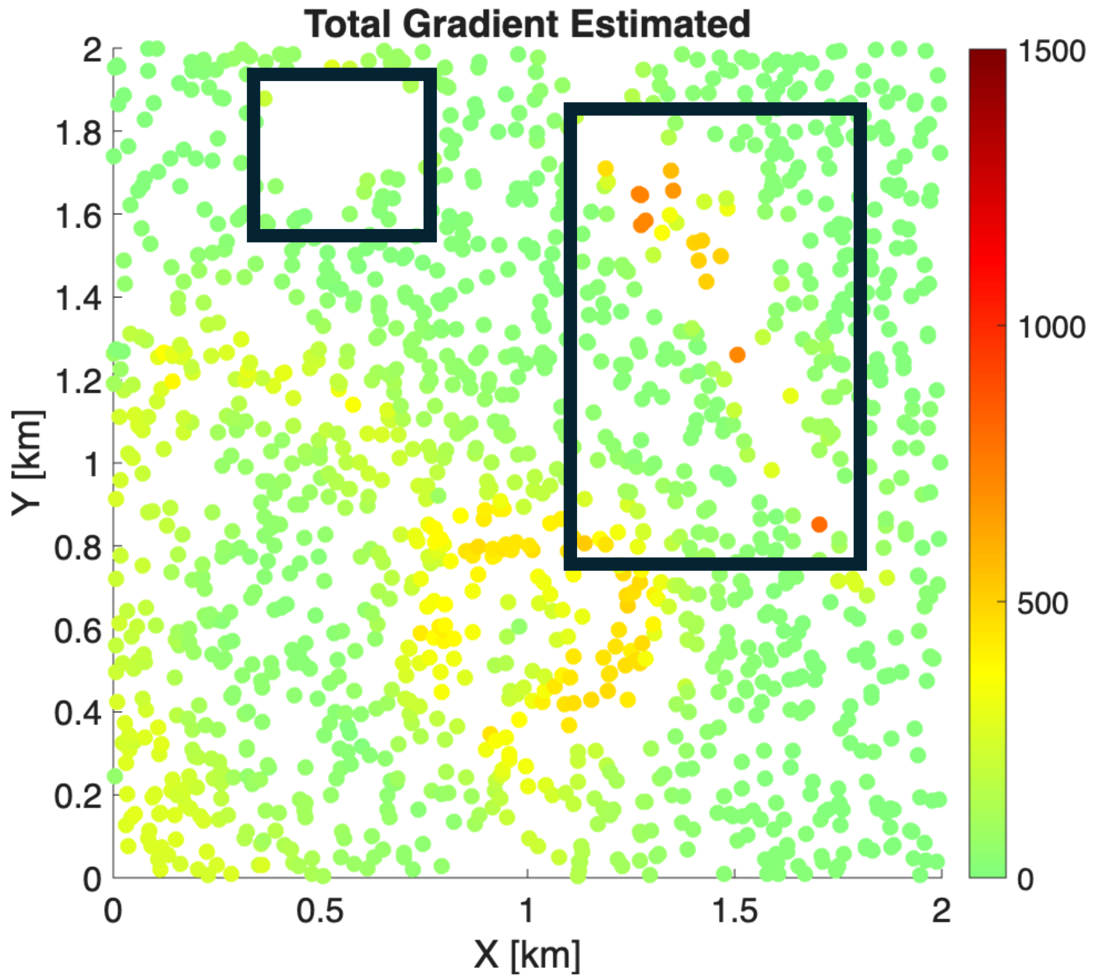


Lets look at some example
Note on the missing points

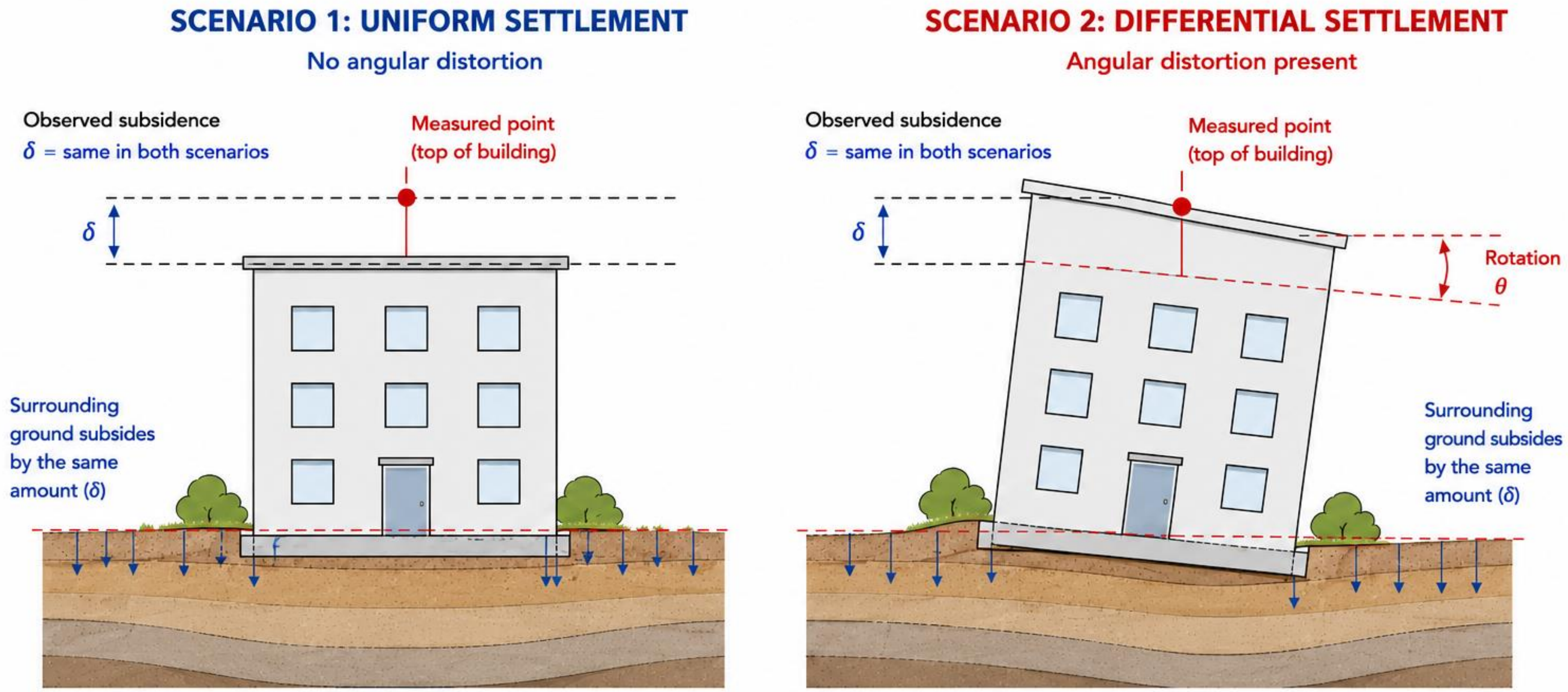
True



Proposed approach (adaptive)



Note on the interpretation of angular distortion



- Building settles uniformly by δ
- Surrounding ground also subsides by δ
- No rotation or angular distortion
- Structure remains undeformed

- Same observed subsidence δ at the top point
- Caused by differential settlement within the structure (and relative to the surrounding ground)
- Building undergoes rotation θ (angular distortion)
- Structure is deformed

KEY MESSAGE:

Identical subsidence observed at **a single point** can correspond to fundamentally different deformation states.
An angular distortion parameter estimated from a single point is **insufficient** to characterize the building's actual angular distortion.

Main Conclusions

- **Adaptive multi-scale estimation** enables more optimal differential settlement mapping from InSAR data.
- **Error-aware processing** allows uncertainty and quality assessment of the estimated gradients.
- **Computationally efficient**, making it suitable for large-scale hazard screening
- **Results should be interpreted as differential settlement, not necessarily angular distortion.**
- **Angular distortion requires multiple measurements on a single structure.**
- **With typical InSAR resolution, estimated gradients cannot be directly translated into building stress or damage (except for high resolution data).**
- **Use InSAR-derived differential settlement products with appropriate caution.**