Exploiting the Gradient–Direction Constraint for 3D Decomposition of InSAR LOS Deformation in Subsidence Areas

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Some Recaps: InSAR LOS Decomposition

- For one track and for each pixel: One equation and three unknown

$$d_{LOS} = \begin{bmatrix} -\sin\theta\cos\alpha & \sin\theta\sin\alpha & \cos\theta \end{bmatrix} \begin{bmatrix} d_e \\ d_n \\ d_u \end{bmatrix}$$

- For two tracks and for each pixel: two equations and three unknown

$$\begin{bmatrix} d_{asc} \\ d_{dsc} \end{bmatrix} = \begin{bmatrix} -\sin\theta_a\cos\alpha_a & \sin\theta_a\sin\alpha_a & \cos\theta_a \\ -\sin\theta_d\cos\alpha_d & \sin\theta_d\sin\alpha_d & \cos\theta_d \end{bmatrix} \begin{bmatrix} d_e \\ d_n \\ d_u \end{bmatrix}$$

- We still need one more constrain to solve the system:

1. a common practice: Ignoring the north component (Logic: InSAR has low sensitivity to the north component)
2. or somehow put another constrain on the direction of the horizontal deformation

InSAR LOS Decomposition:
Error that we make by ignoring the north component

Observation-equation: $$\begin{bmatrix} d_{asc} \\ d_{dsc} \end{bmatrix} = \begin{bmatrix} -\sin\theta_a \cos\alpha_a & \cos\theta_a \\ -\sin\theta_d \cos\alpha_d & \cos\theta_d \end{bmatrix} \begin{bmatrix} d_e \\ d_u \end{bmatrix}$$

Solution: $$\begin{bmatrix} \hat{d}_e \\ \hat{d}_u \end{bmatrix} = \begin{bmatrix} -\sin\theta_a \cos\alpha_a & \cos\theta_a \\ -\sin\theta_d \cos\alpha_d & \cos\theta_d \end{bmatrix}^{-1} \begin{bmatrix} d_{asc} \\ d_{dsc} \end{bmatrix}$$

Error that we make (bias): $$\boxed{\begin{bmatrix} E_e \\ E_u \end{bmatrix} = \begin{bmatrix} \hat{d}_e - d_{asc} \\ \hat{d}_u - d_{dsc} \end{bmatrix}}$$

InSAR LOS Decomposition:
Error that we make by ignoring the north component

Observation-equation: $$\begin{bmatrix} d_{asc} \\ d_{dsc} \end{bmatrix} = \begin{bmatrix} -\sin\theta_a\cos\alpha_a & \cos\theta_a \\ -\sin\theta_d\cos\alpha_d & \cos\theta_d \end{bmatrix} \begin{bmatrix} d_e \\ d_u \end{bmatrix}$$

Solution: $$\begin{bmatrix} \hat{d}_e \\ \hat{d}_u \end{bmatrix} = \begin{bmatrix} -\sin\theta_a\cos\alpha_a & \cos\theta_a \\ -\sin\theta_d\cos\alpha_d & \cos\theta_d \end{bmatrix}^{-1} \begin{bmatrix} d_{asc} \\ d_{dsc} \end{bmatrix}$$

Error that we make (bias): $$\boxed{\begin{bmatrix} E_e \\ E_u \end{bmatrix} = \begin{bmatrix} \hat{d}_e - d_{asc} \\ \hat{d}_u - d_{dsc} \end{bmatrix}}$$

$$E_u = -\frac{\sin(\theta_a)\sin(\theta_d)\sin(\alpha_a - \alpha_d)}{\cos(\alpha_a)\cos(\theta_d)\sin(\theta_a) - \cos(\alpha_d)\cos(\theta_a)\sin(\theta_d)} d_n$$

$$E_e = -\frac{\sin(\alpha_a)\cos(\theta_d)\sin(\theta_a) - \sin(\alpha_d)\cos(\theta_a)\sin(\theta_d)}{\cos(\alpha_a)\cos(\theta_d)\sin(\theta_a) - \cos(\alpha_d)\cos(\theta_a)\sin(\theta_d)} d_n$$

InSAR LOS Decomposition:
Error that we make by ignoring the north component

$$E_u = -\frac{\sin(\theta_a)\sin(\theta_d)\sin(\alpha_a - \alpha_d)}{\cos(\alpha_a)\cos(\theta_d)\sin(\theta_a) - \cos(\alpha_d)\cos(\theta_a)\sin(\theta_d)} d_n$$

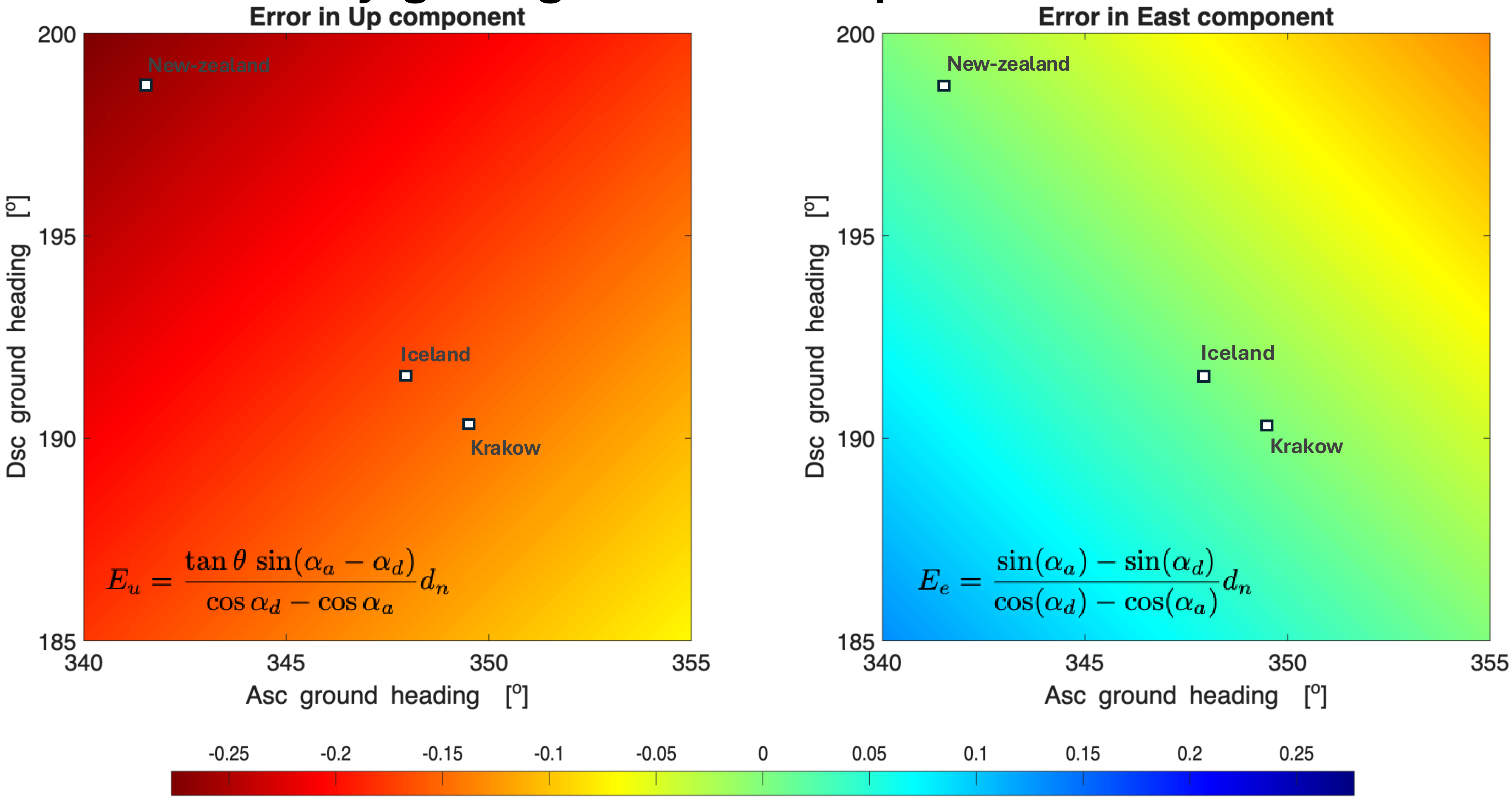
$$E_e = -\frac{\sin(\alpha_a)\cos(\theta_d)\sin(\theta_a) - \sin(\alpha_d)\cos(\theta_a)\sin(\theta_d)}{\cos(\alpha_a)\cos(\theta_d)\sin(\theta_a) - \cos(\alpha_d)\cos(\theta_a)\sin(\theta_d)} d_n$$

Assuming same inc angle for both tracks:

$$E_u = \frac{\tan\theta\,\sin(\alpha_a - \alpha_d)}{\cos\alpha_d - \cos\alpha_a} d_n \qquad E_e = \frac{\sin(\alpha_a) - \sin(\alpha_d)}{\cos(\alpha_d) - \cos(\alpha_a)} d_n$$

Error that we make by ignoring the north component

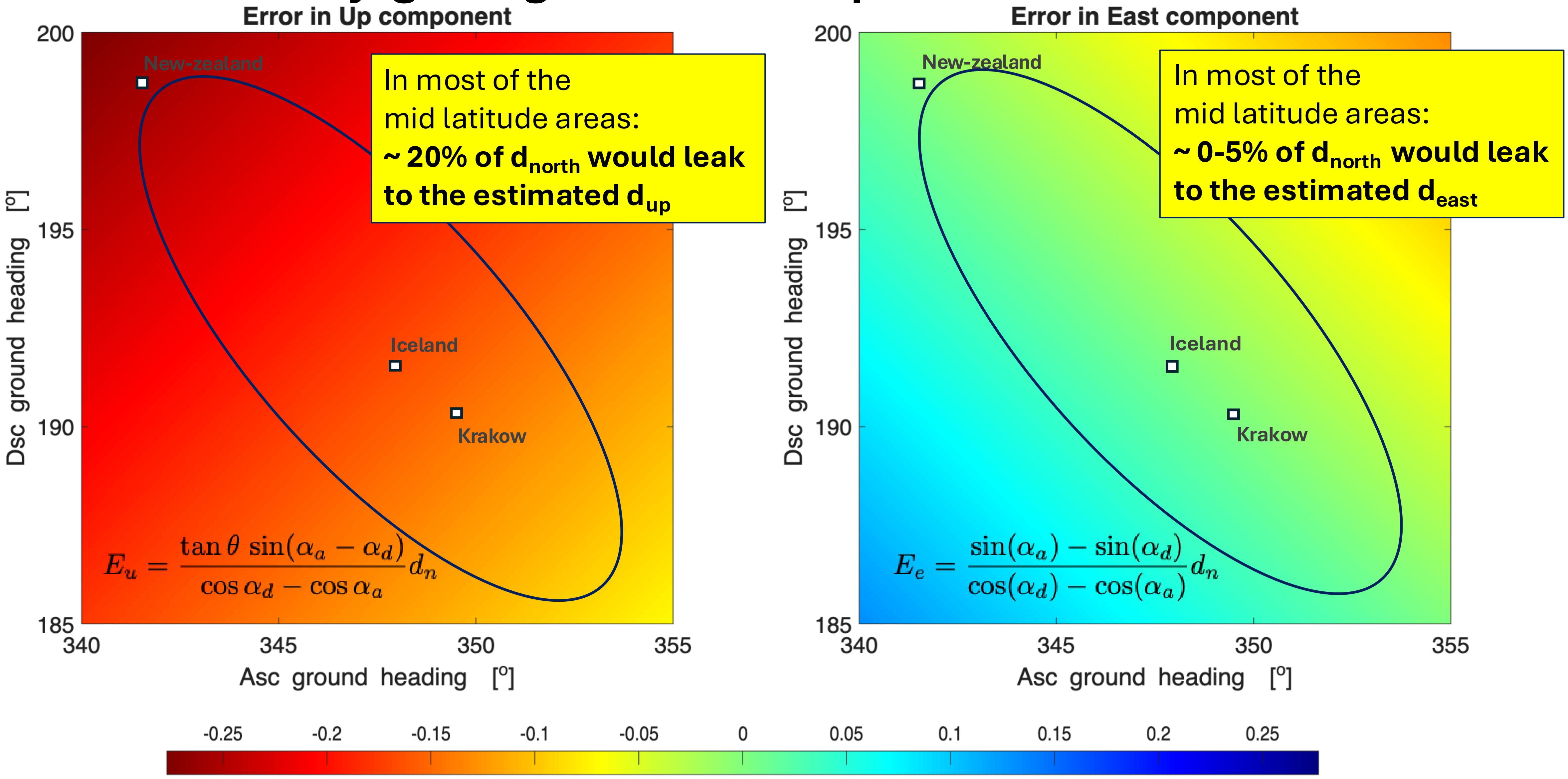
For inc=40 degree



Fraction of the deformation north component that leaks to the up and east

Error that we make by ignoring the north component

For inc=40 degree



Fraction of the deformation north component that leaks to the up and east

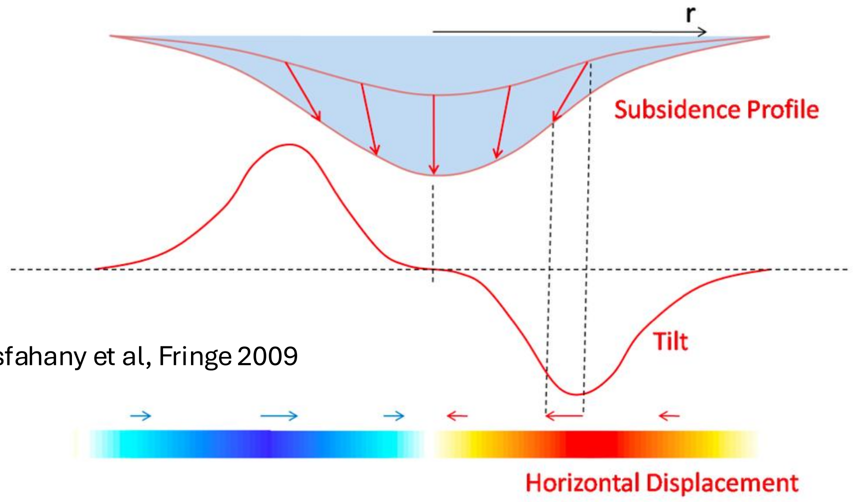
InSAR LOS Decomposition: Gradient–Direction Constraint

- If we know the direction of the horizontal component γ
- Two equations and two unknowns (solvable)

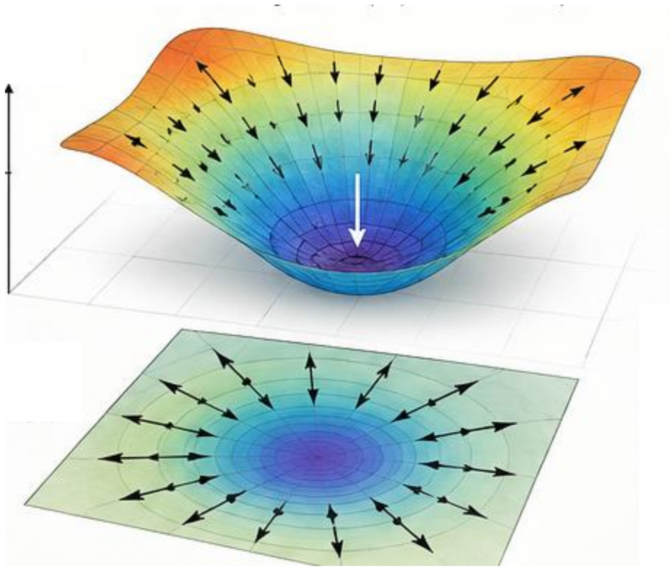
$$\begin{bmatrix} d_a \\ d_d \end{bmatrix} = \begin{bmatrix} \cos(\theta_a) & \sin(\theta_a)\sin(\alpha_a - \gamma) \\ \cos(\theta_d) & \sin(\theta_d)\sin(\alpha_d - \gamma) \end{bmatrix} \begin{bmatrix} d_u \\ d_\gamma \end{bmatrix}$$

- How can we somehow estimate this direction ???

InSAR LOS Decomposition: Gradient–Direction Constraint



Samiei-Esfahany et al, Fringe 2009



Gradient–Direction Constraint

The horizontal gradient of vertical deformation is aligned with the direction of horizontal deformation.

$$\arctan\left(\frac{\partial d_u/\partial n}{\partial d_u/\partial e}\right) = \arctan\left(\frac{d_n}{d_e}\right)$$

Does this condition always hold?
If it does, we can use it for full 3D InSAR decomposition.

- Under what conditions is it valid?
- If valid, how can we compute the tilt initially ?
- If valid and we have a tilt, can precisely estimate all the three components?

InSAR LOS Decomposition: Gradient–Direction Constraint

- **Under what conditions is it valid?**

Assumption

$$\boxed{\nabla_h d_u \parallel d_h} \quad \Rightarrow \quad \tan(\phi_{\nabla d_u}) = \tan(\phi_h)$$

Mogi displacement field

$$d_e = C\frac{e}{R^3}, \qquad d_n = C\frac{n}{R^3}, \qquad d_u = C\frac{u}{R^3} \qquad R = \sqrt{e^2 + n^2 + u^2}, \qquad C = a^3 \Delta P \frac{1-\nu}{\mu}$$

Horizontal gradient of vertical deformation

$$\frac{\partial d_u}{\partial e} = -\frac{3Cue}{R^5}, \qquad \frac{\partial d_u}{\partial n} = -\frac{3Cun}{R^5} \qquad \Rightarrow \qquad \frac{\partial d_u/\partial n}{\partial d_u/\partial e} = \frac{n}{e}$$

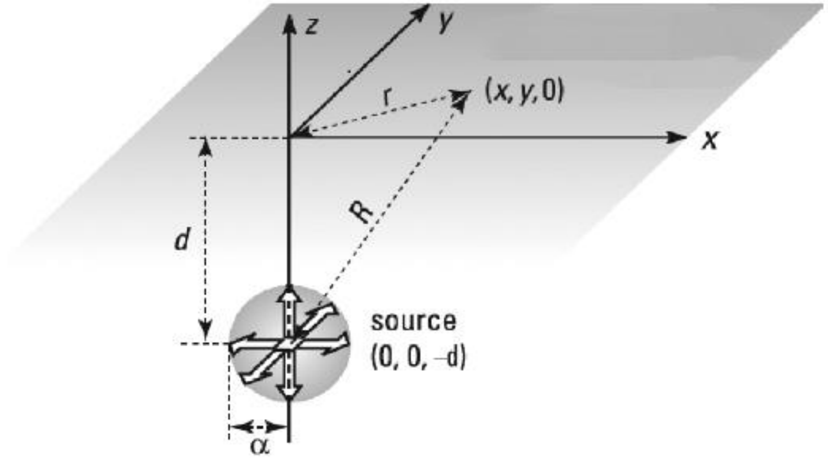
Horizontal displacement direction

$$\frac{d_n}{d_e} = \frac{Cn/R^3}{Ce/R^3} = \frac{n}{e} \qquad \Rightarrow \qquad \frac{d_n}{d_e} = \frac{n}{e}$$

Result

$$\boxed{\frac{\partial d_u/\partial n}{\partial d_u/\partial e} = \frac{d_n}{d_e}} \quad \Rightarrow \quad \phi_{\nabla d_u} = \phi_h$$

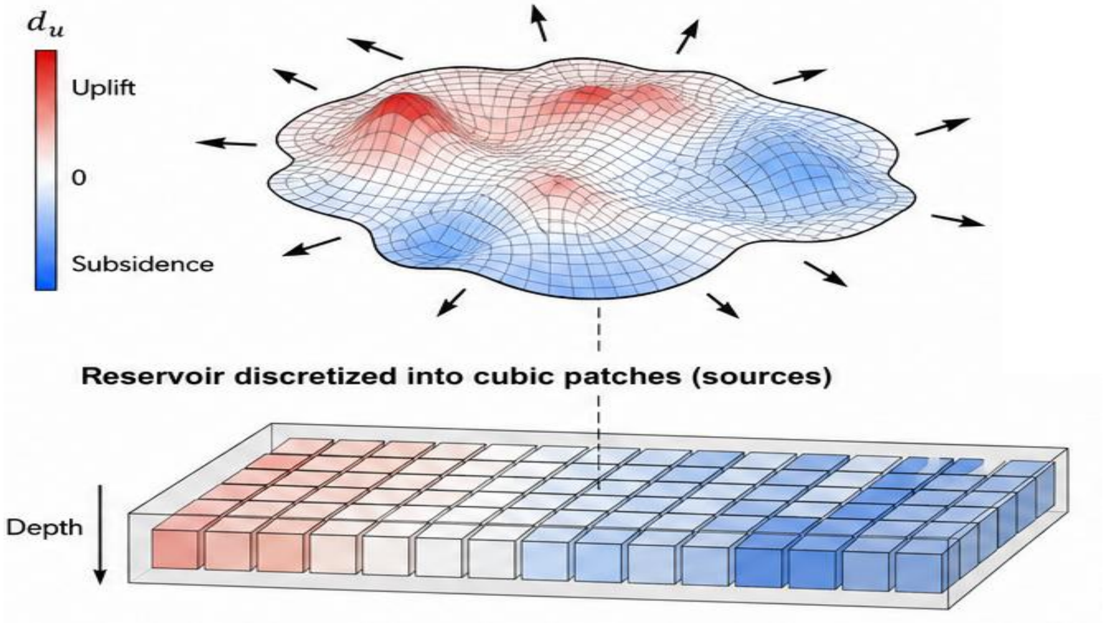
For a Mogi point source, the horizontal gradient of vertical deformation is aligned with the direction of horizontal deformation.



Simple Mogi point source (volumetric strain)

InSAR LOS Decomposition: Gradient–Direction Constraint

- What about more complicated sources



$$d_e = \sum_i d_{e,i}, \qquad d_n = \sum_i d_{n,i}, \qquad d_u = \sum_i d_{u,i}$$

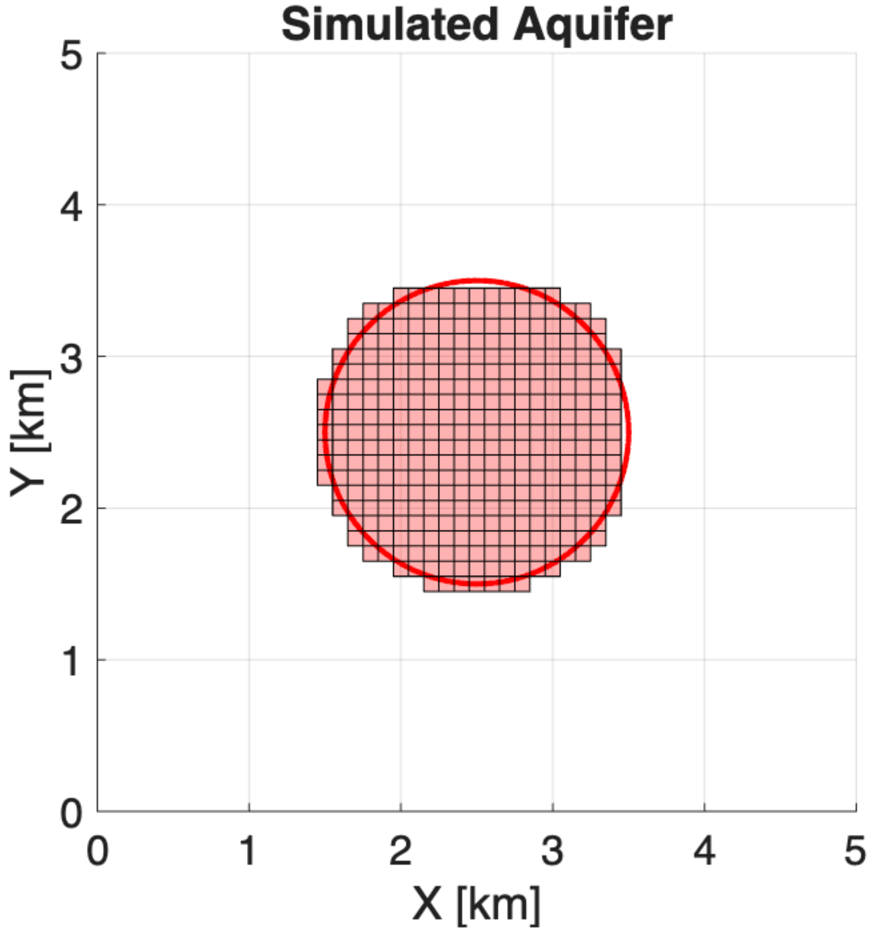
$$\boxed{\frac{\sum_i \partial d_{u,i}/\partial n}{\sum_i \partial d_{u,i}/\partial e} \neq \frac{\sum_i d_{n,i}}{\sum_i d_{e,i}}}$$

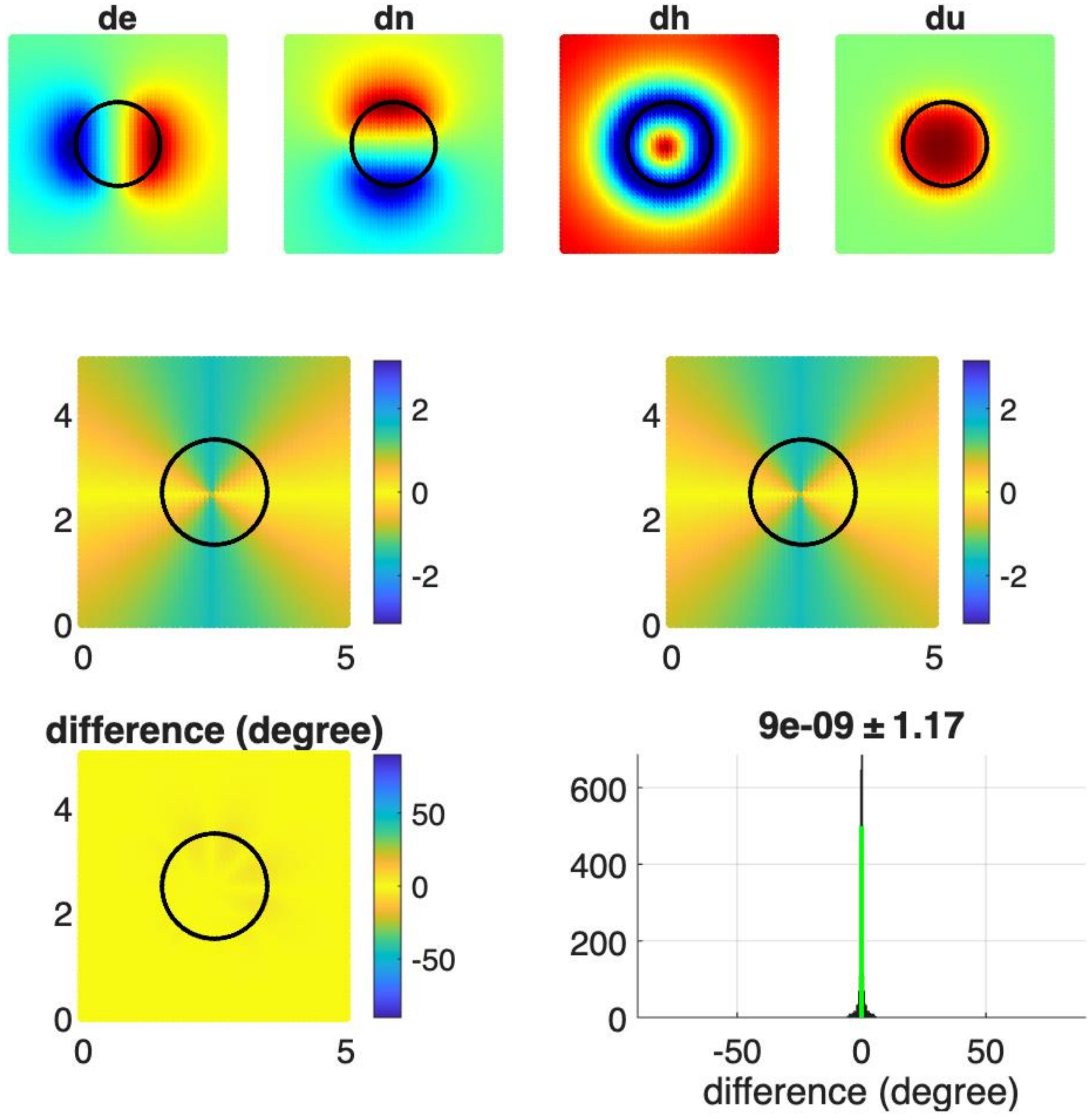
For complex sources, superposition does not necessarily preserve the Gradient–Direction Constraint.

Let's test it via simulation

Simulation 1

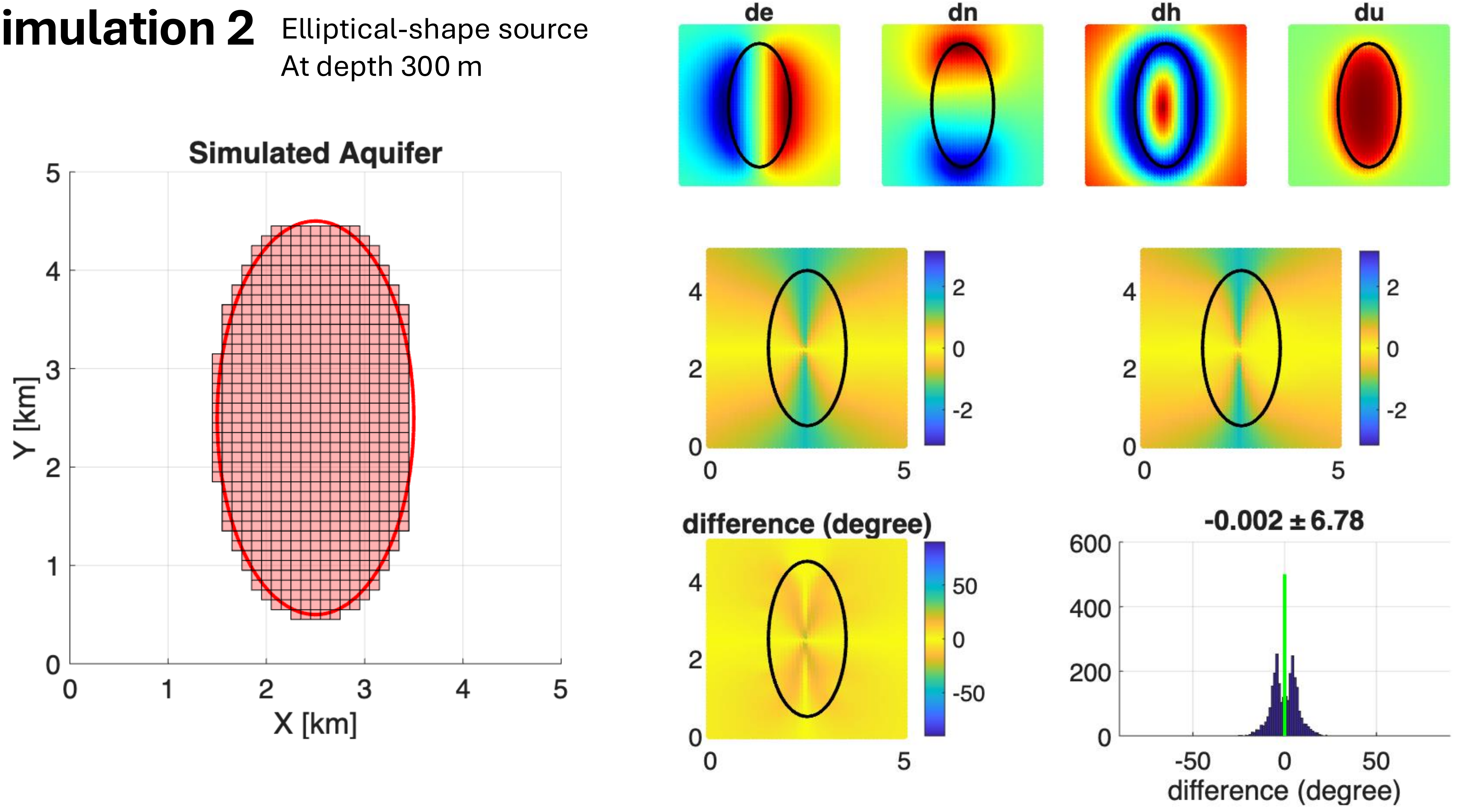
Disk-shape source
At depth 300 m





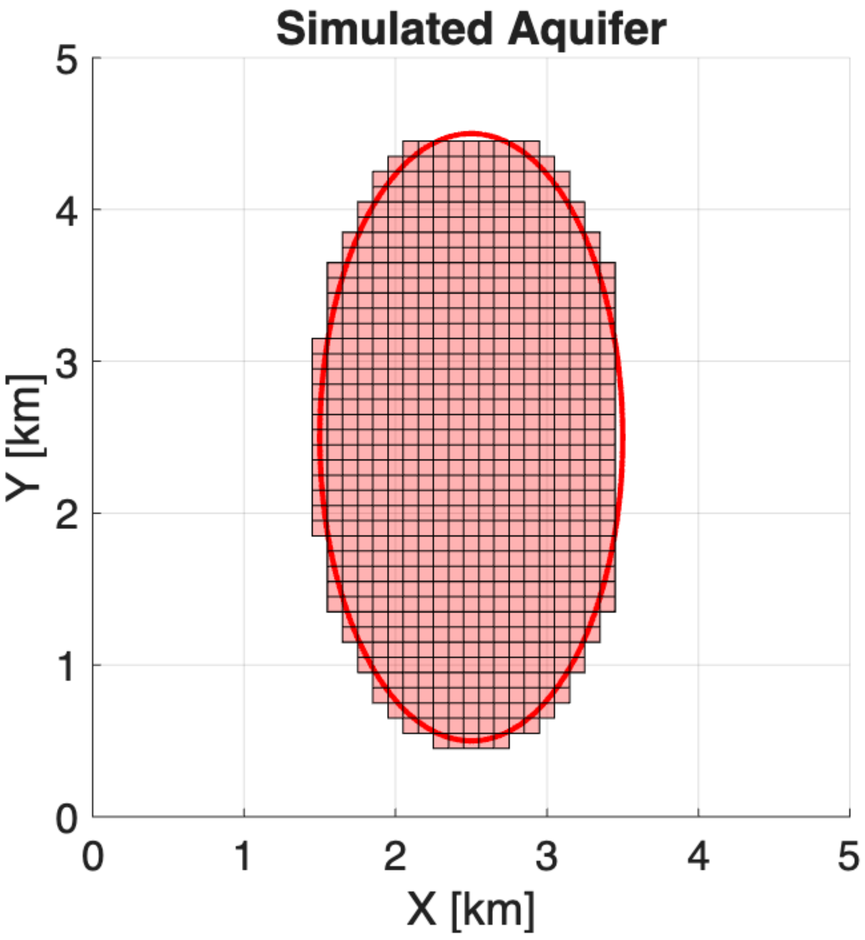
Simulation 2

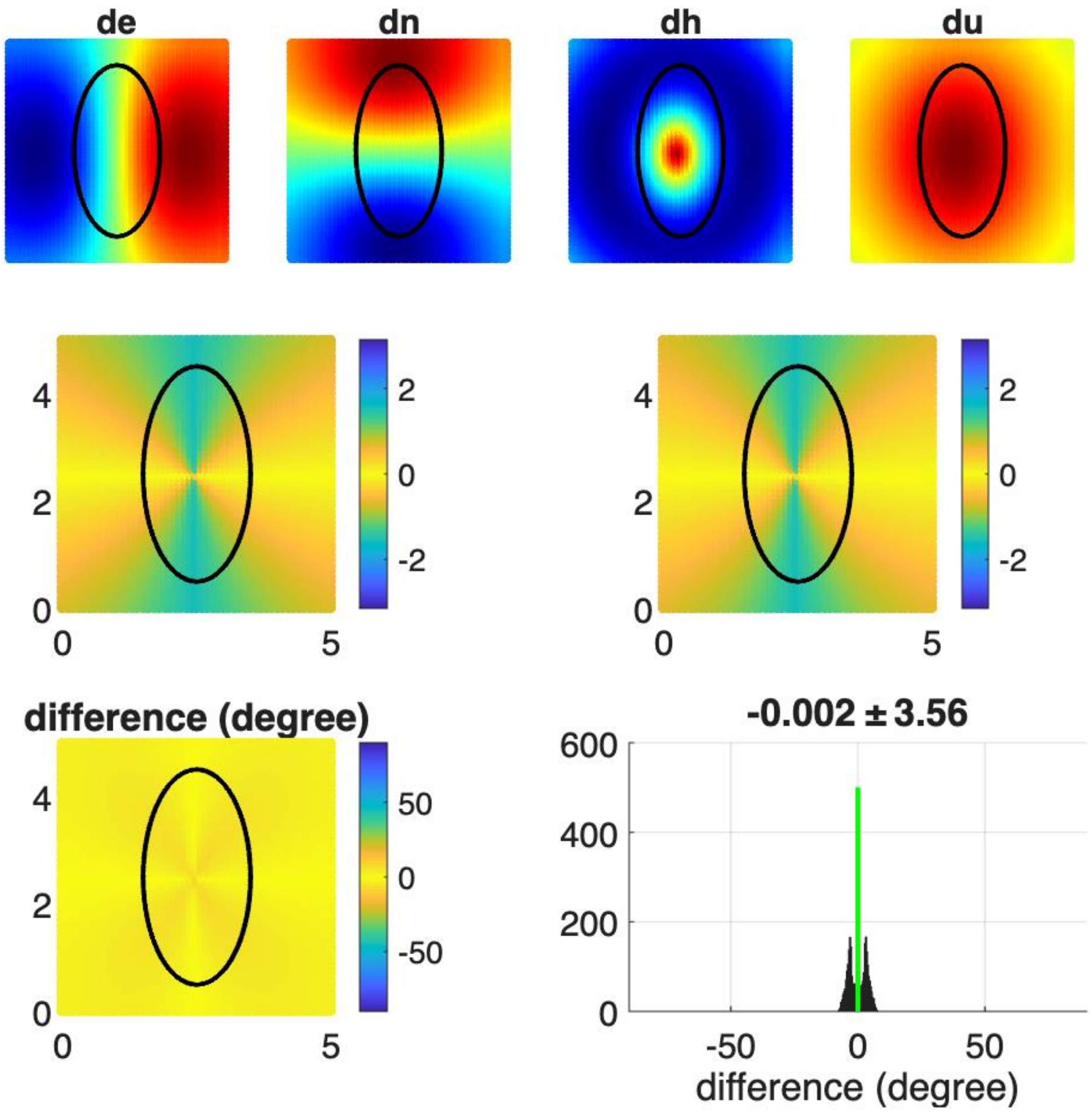
Elliptical-shape source
At depth 300 m



Simulation 1

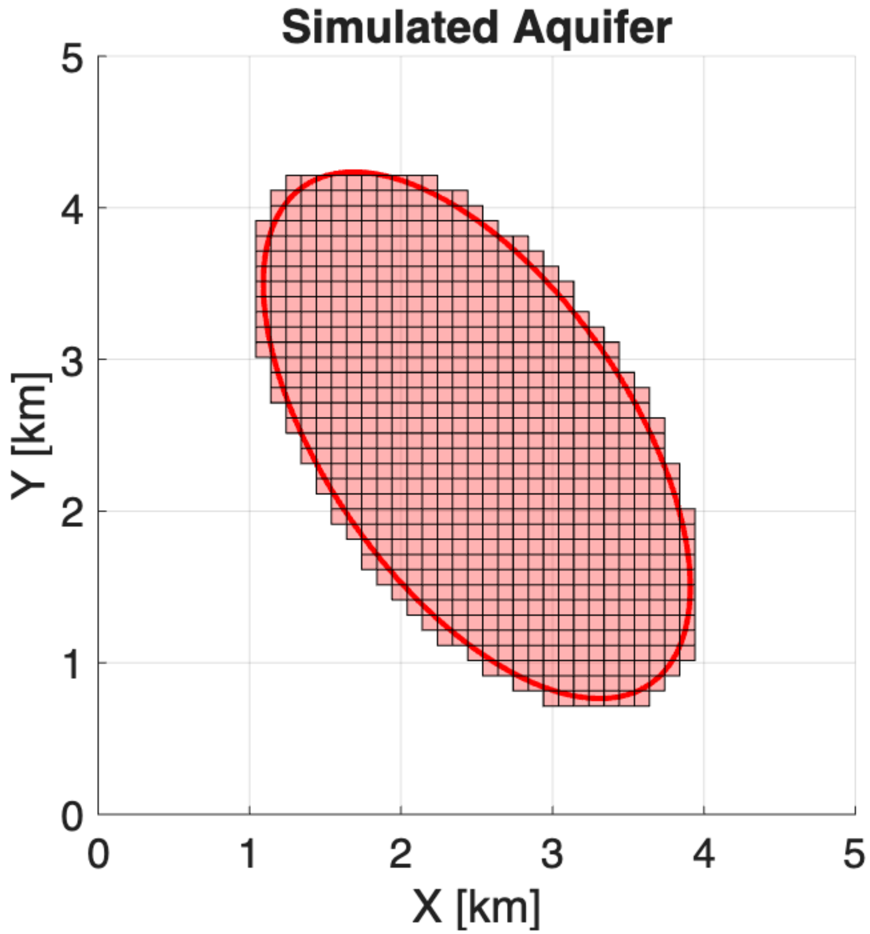
Elliptical-shape source
At depth 2000 m

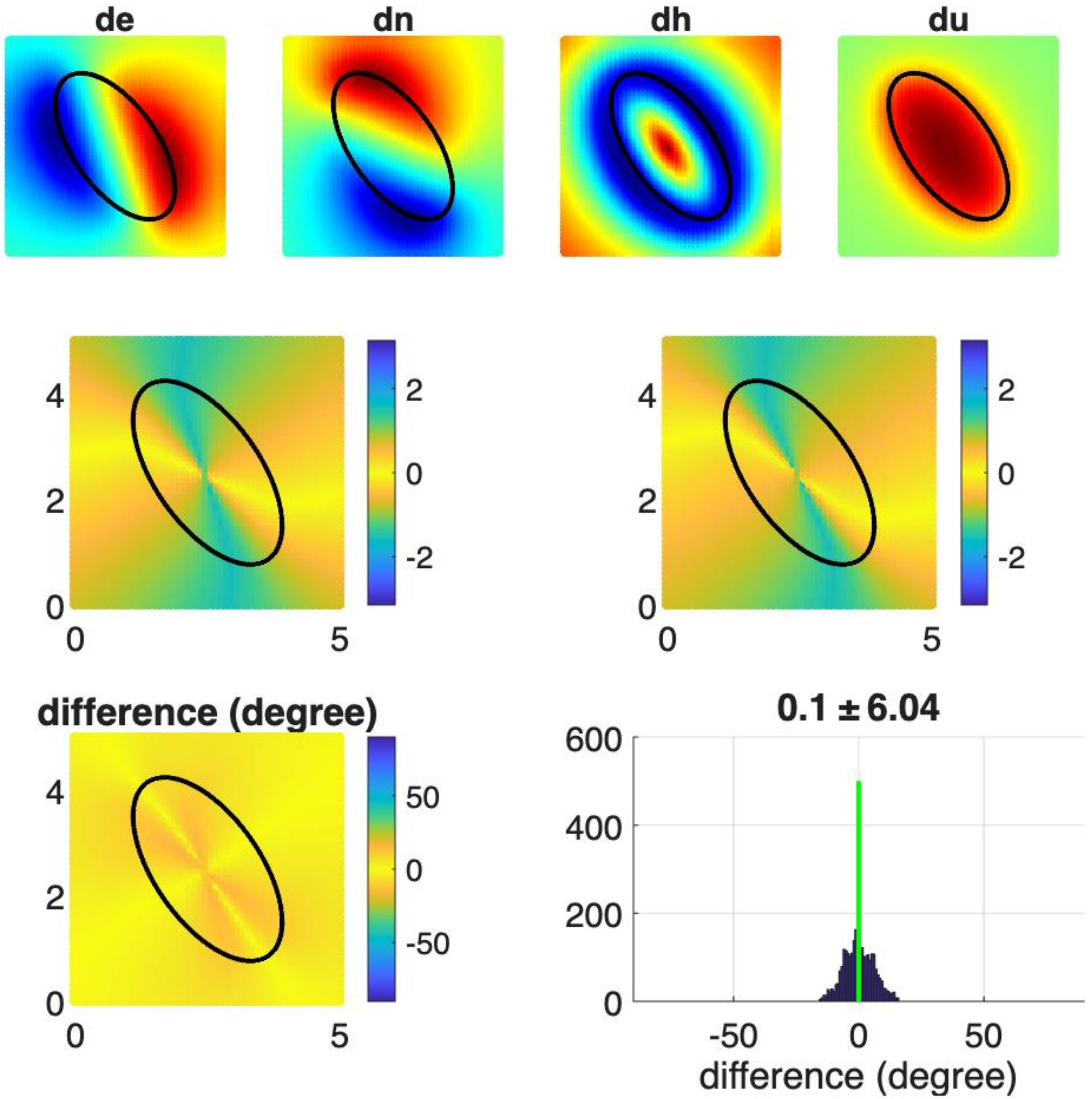




Simulation 1

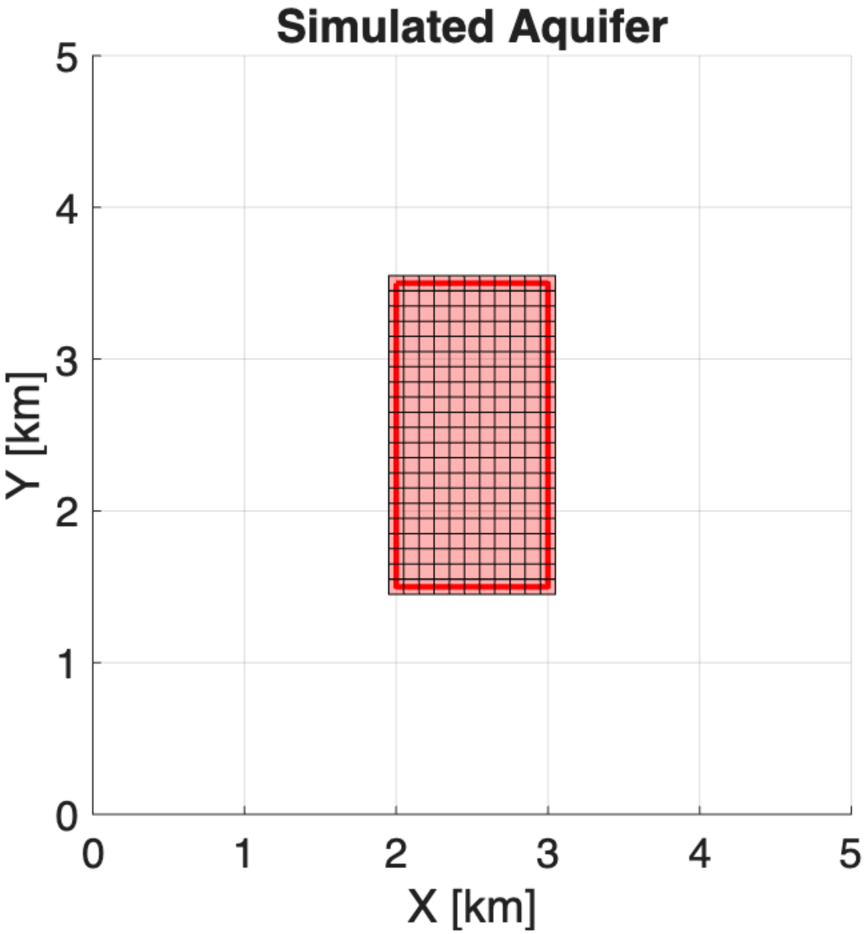
Elliptical-shape source
Rotated
At depth 500 m

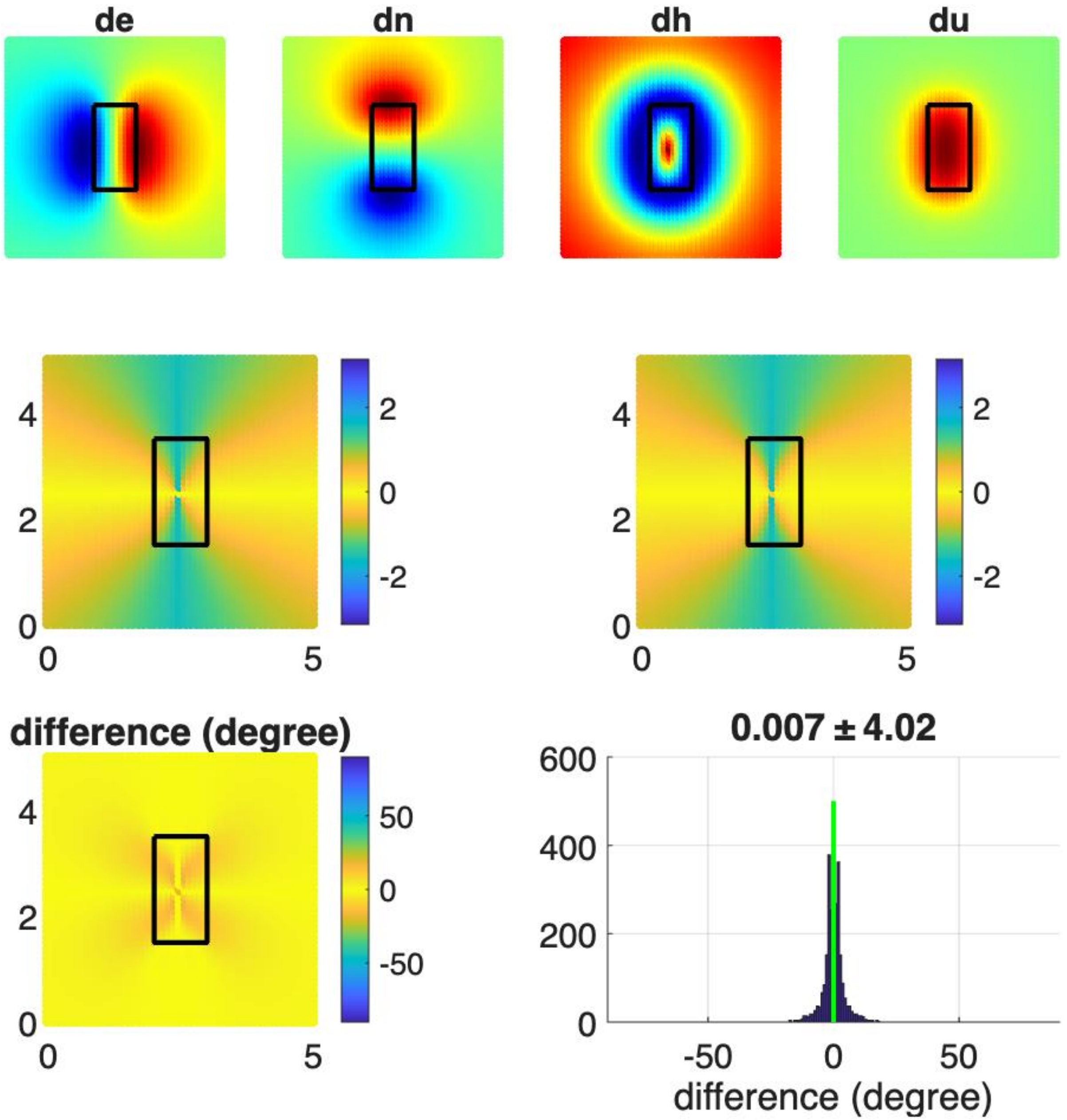




Simulation 1

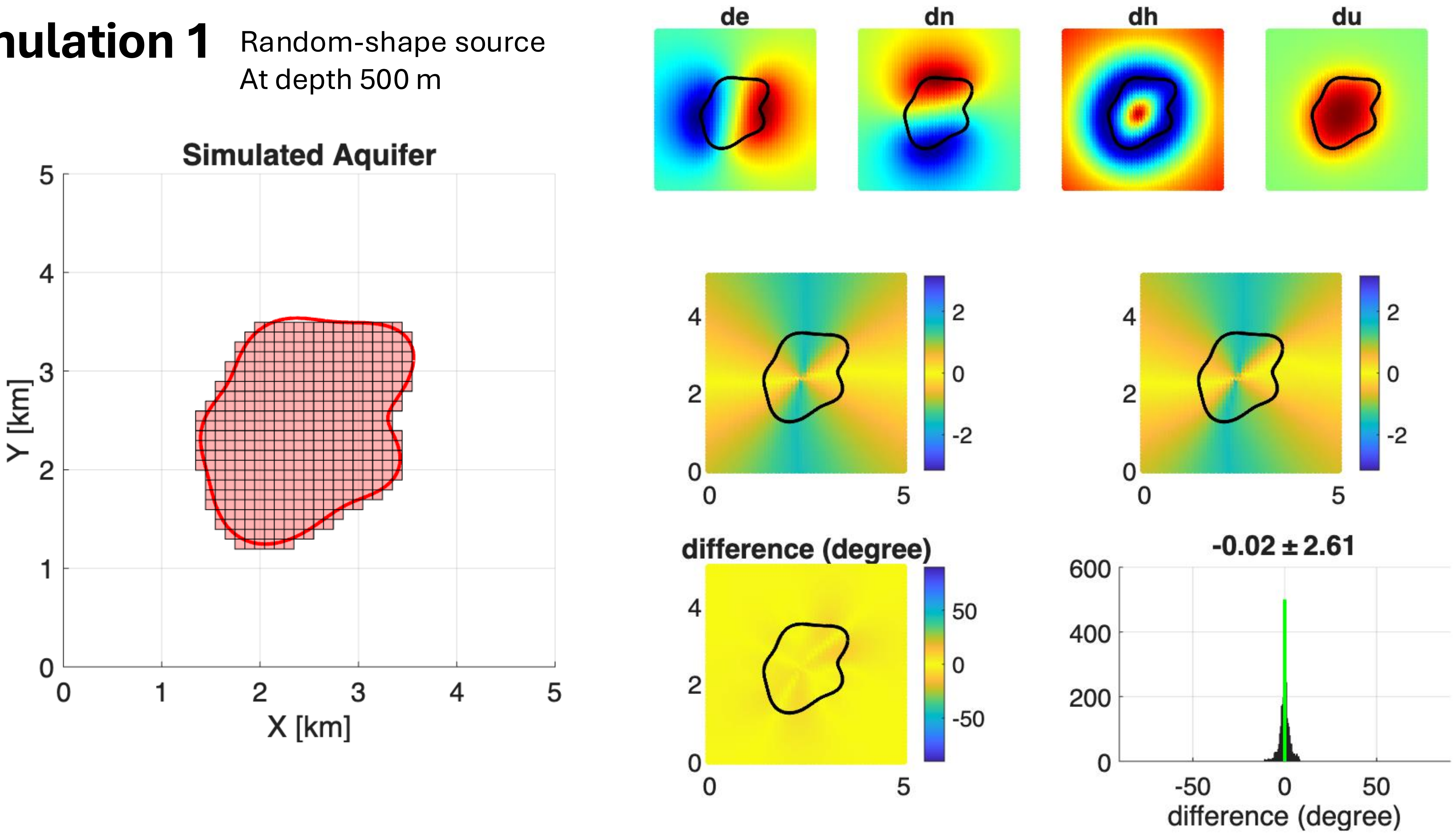
Rectangular source
At depth 500 m





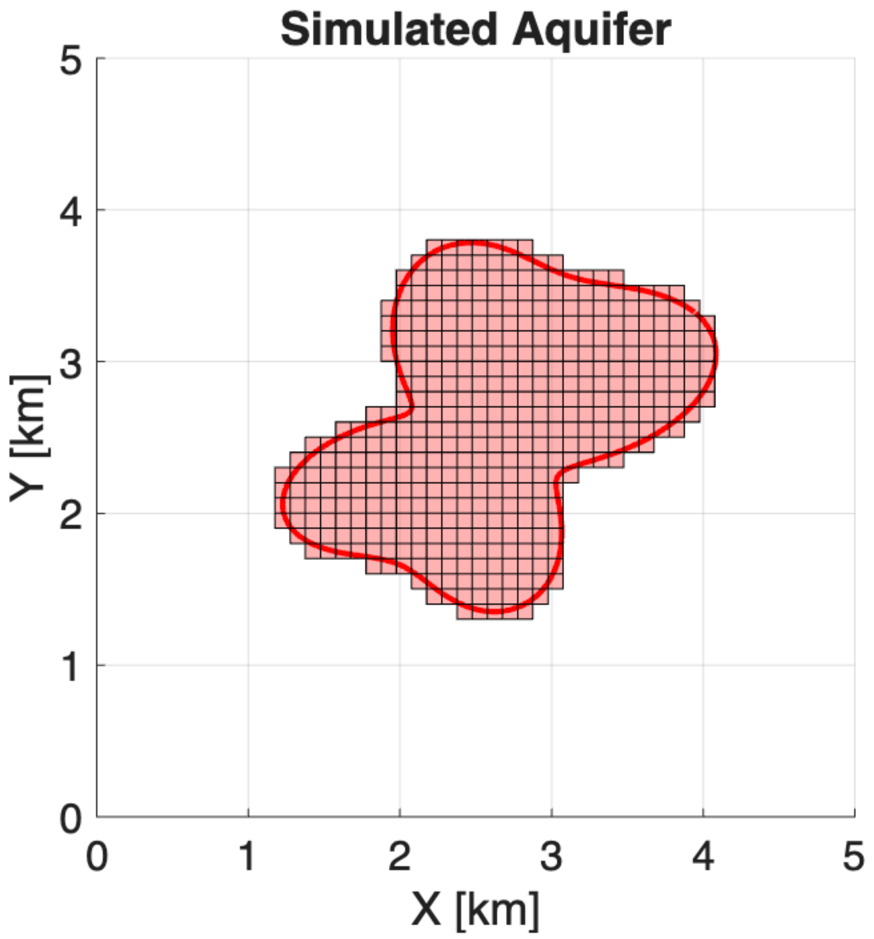
Simulation 1

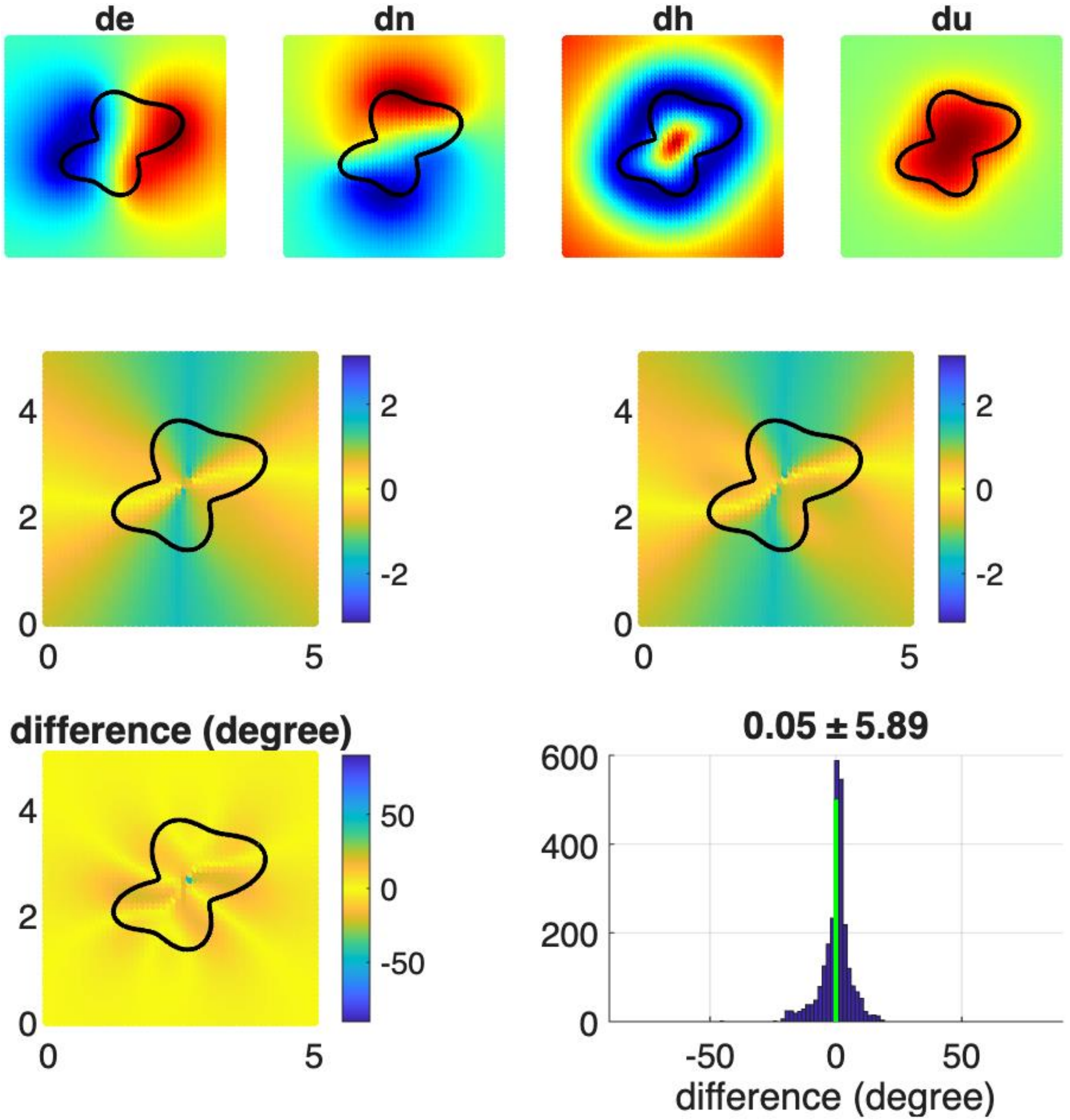
Random-shape source
At depth 500 m



Simulation 1

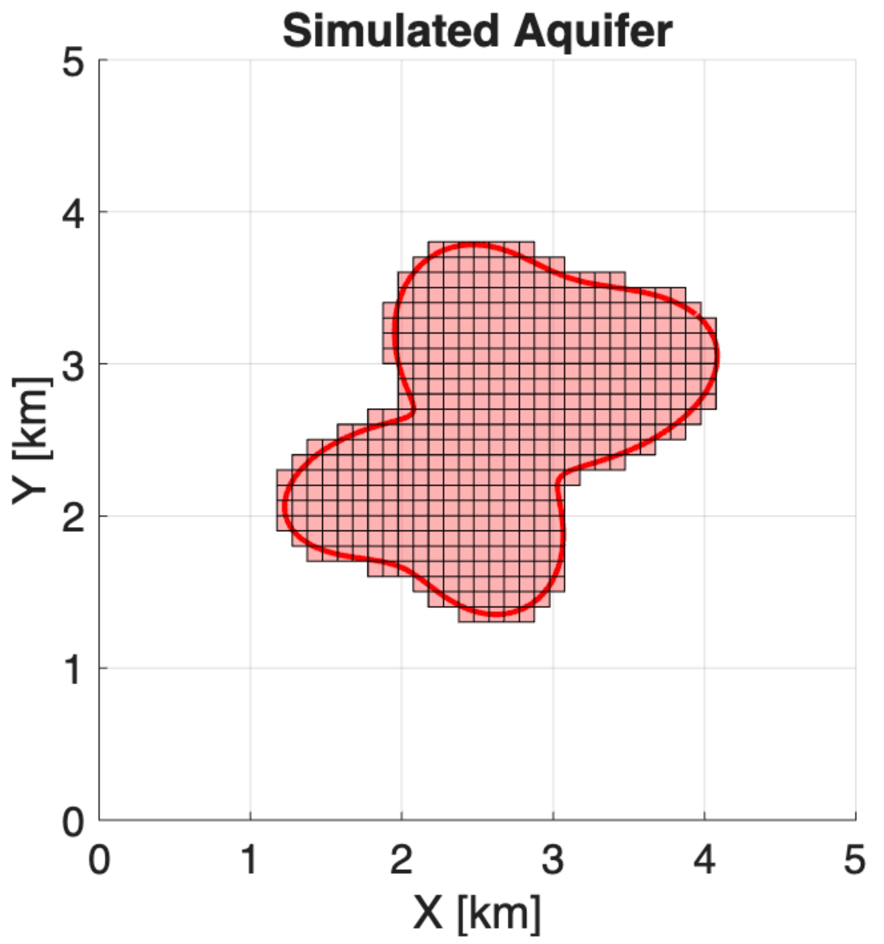
Random-shape source
A bit more complex shape
At depth 500 m

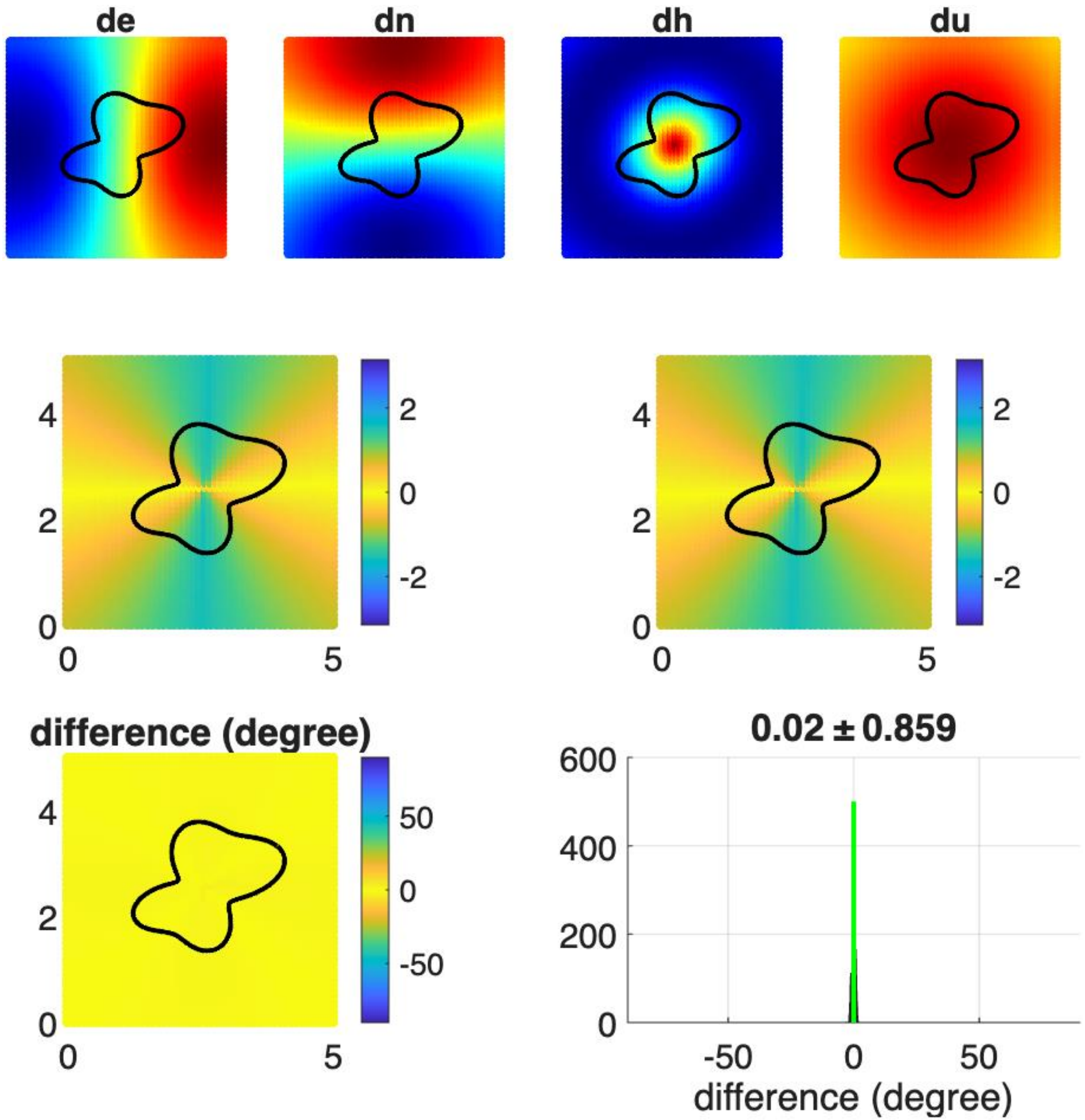




Simulation 1

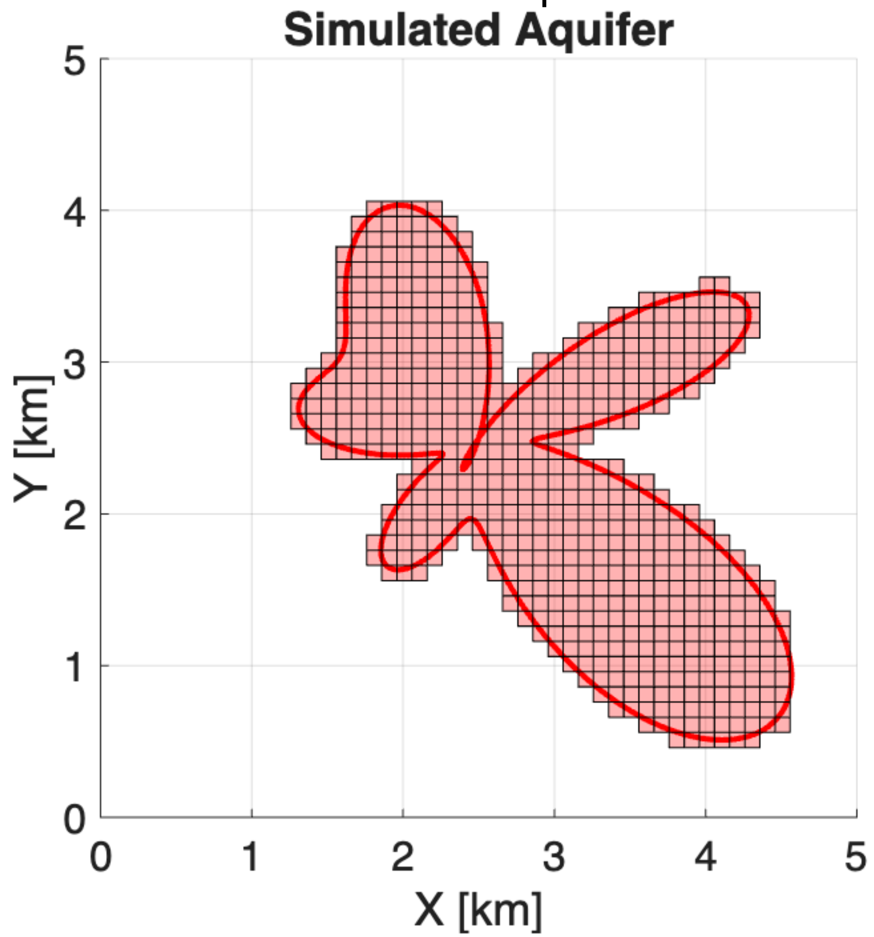
Random-shape source
A bit more complex shape
At depth 3000 m

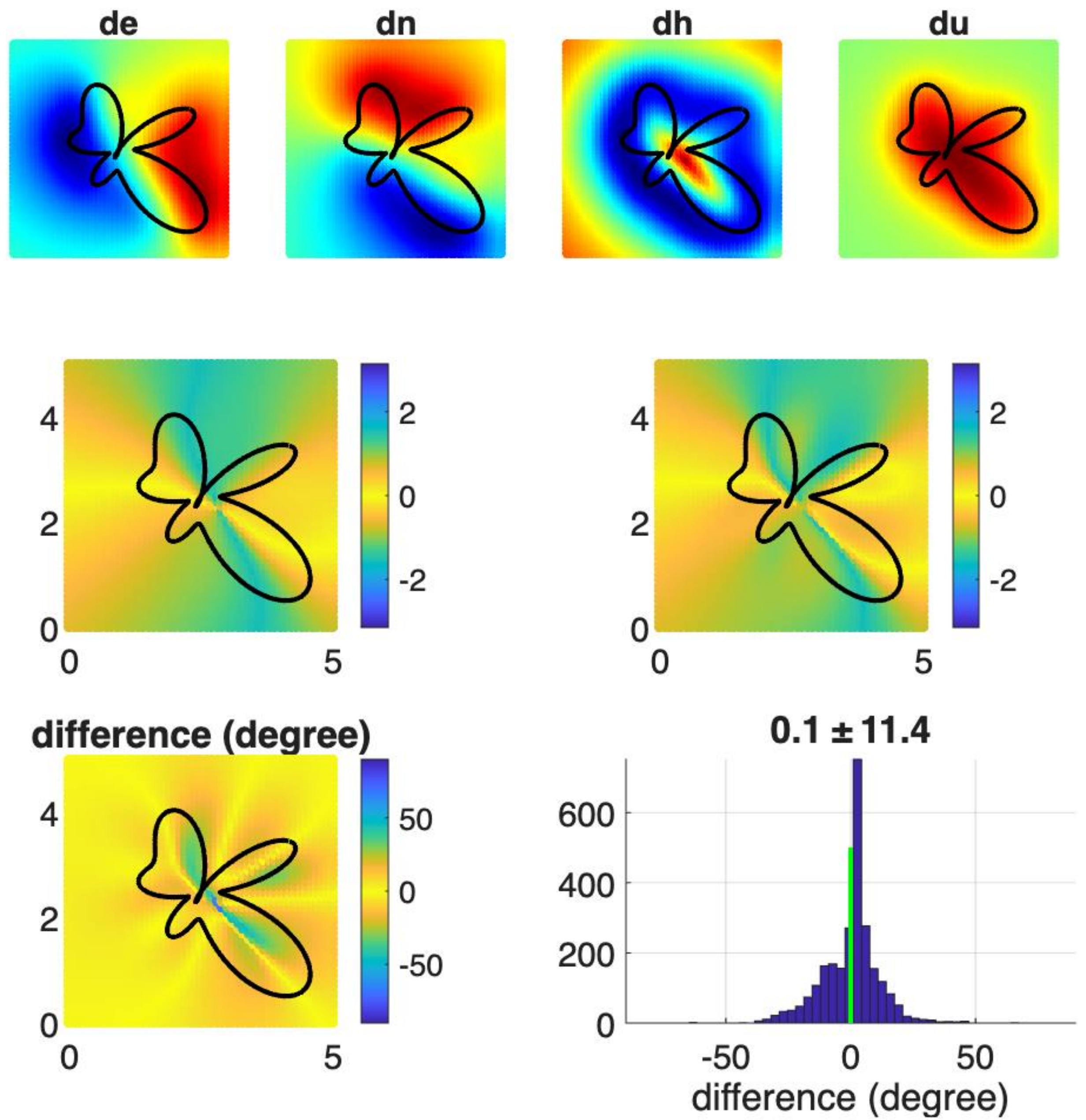




Simulation 1

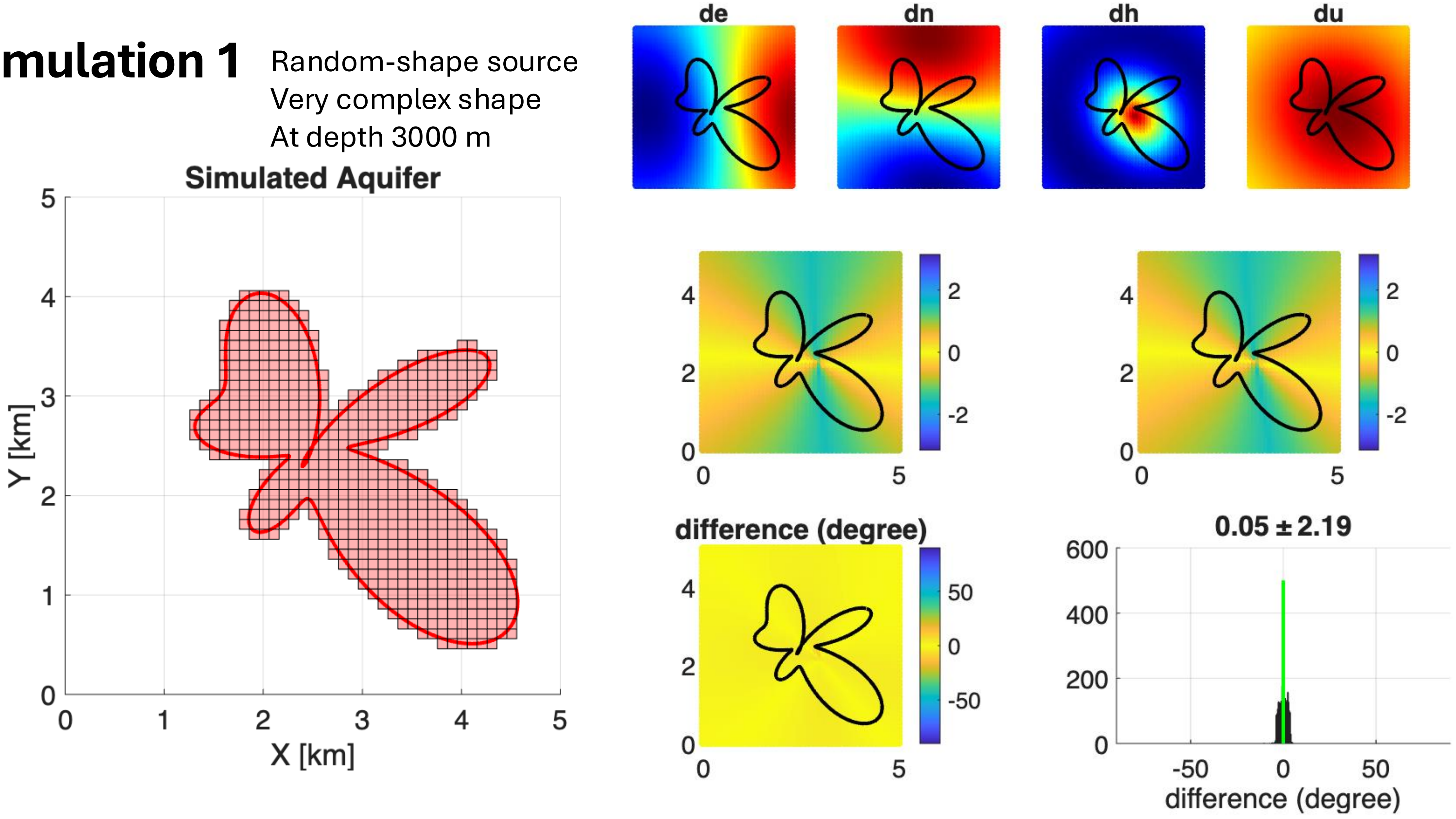
Random-shape source
Very complex shape
At depth 500 m





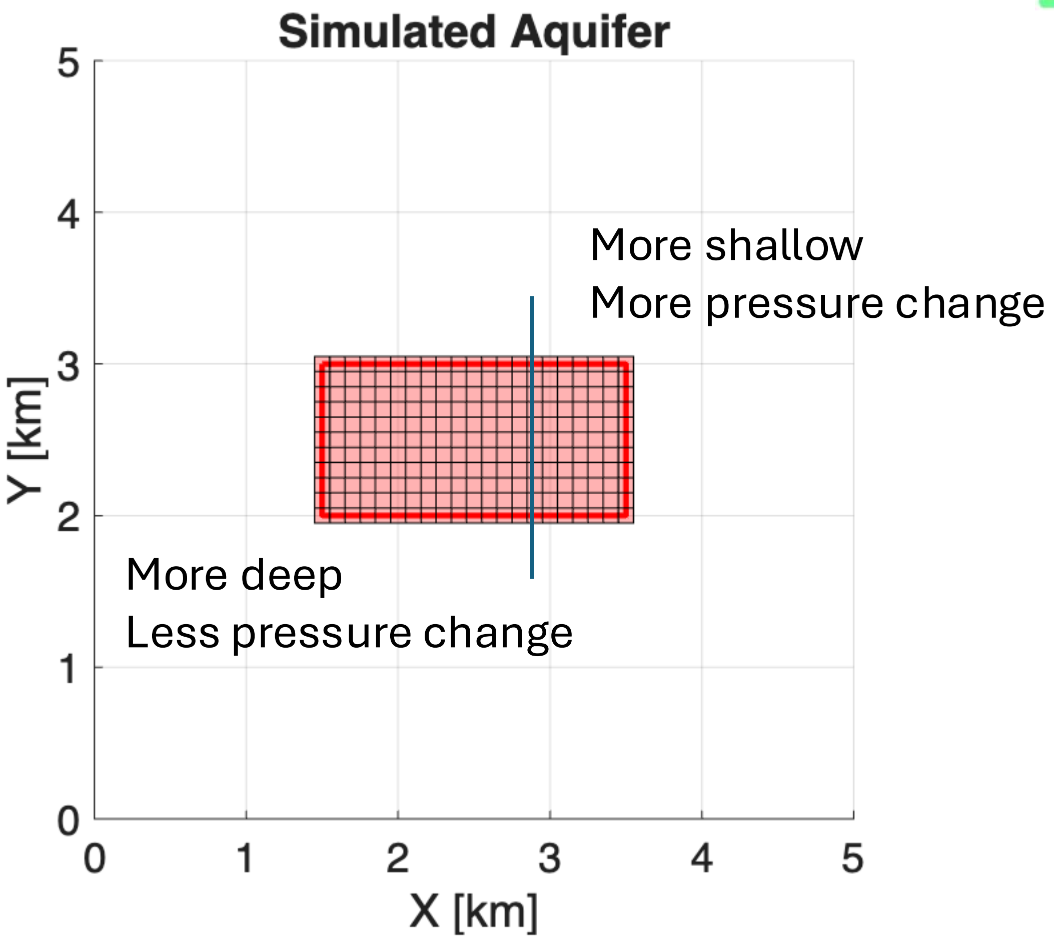
Simulation 1

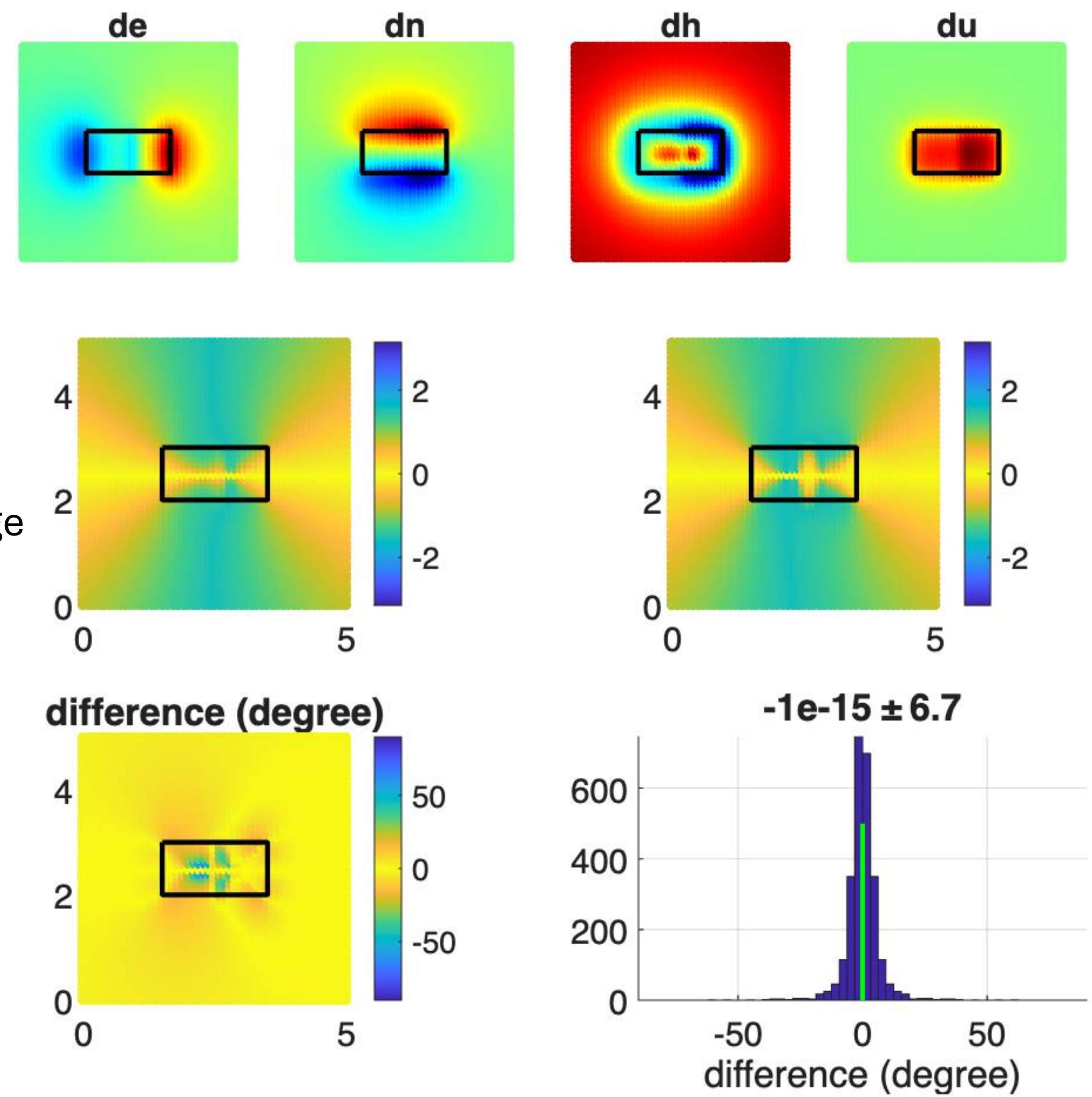
Random-shape source
Very complex shape
At depth 3000 m



Simulation 1

Rectangular source
At depth 500 m
Differing properties
within the source





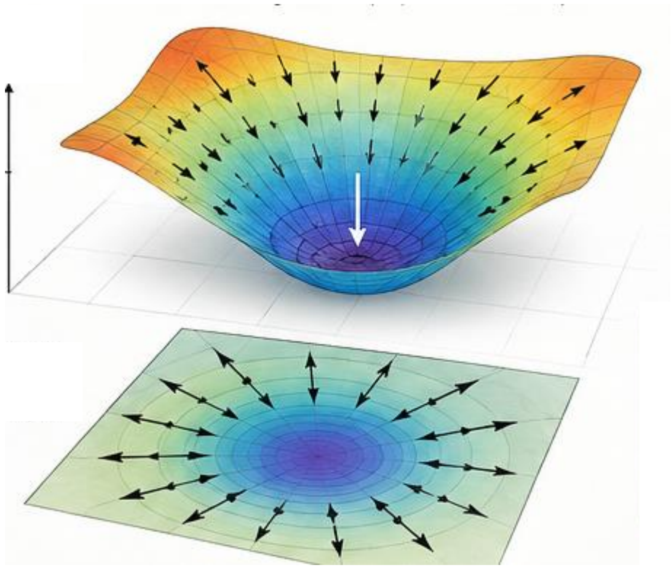
InSAR LOS Decomposition: Gradient–Direction Constraint

Under what conditions is it valid?

Key Findings from Numerical Simulations

✓ **More valid for:**
- Deep sources
- Smooth/simple reservoir geometries
- Deformation dominated by volumetric strain *(hypothesis)*

✗ **Less valid for:**
- Shallow sources
- Complex reservoir geometries
- Strong internal heterogeneity
- Deformation dominated by deviatoric strain *(hypothesis)*

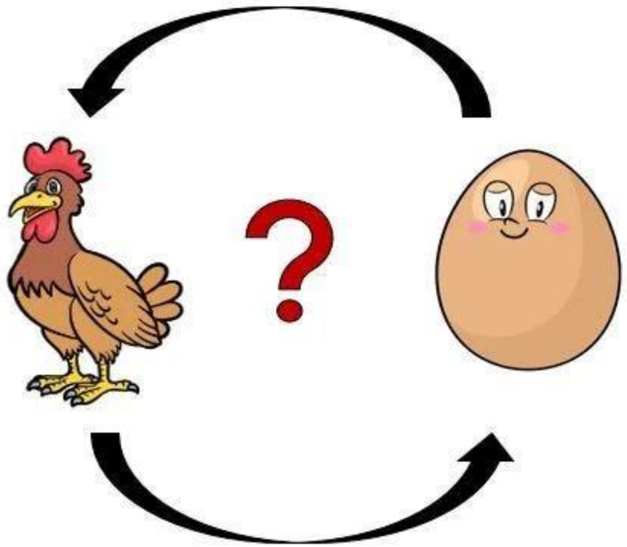


Overall: For many realistic geophysical sources, the misalignment remains within **±10°**, making the assumption suitable for practical applications.

InSAR LOS Decomposition:
How to use the Gradient–Direction Constraint

$$\begin{bmatrix} d_a \\ d_d \end{bmatrix} = \begin{bmatrix} \cos(\theta_a) & \sin(\theta_a)\sin(\alpha_a - \gamma) \\ \cos(\theta_d) & \sin(\theta_d)\sin(\alpha_d - \gamma) \end{bmatrix} \begin{bmatrix} d_u \\ d_\gamma \end{bmatrix}$$

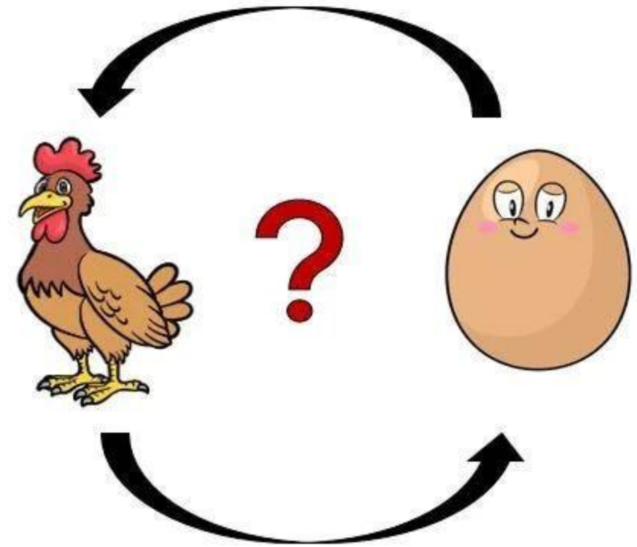
$$\gamma = \arctan\left(\frac{\partial d_u / \partial n}{\partial d_u / \partial e}\right)$$



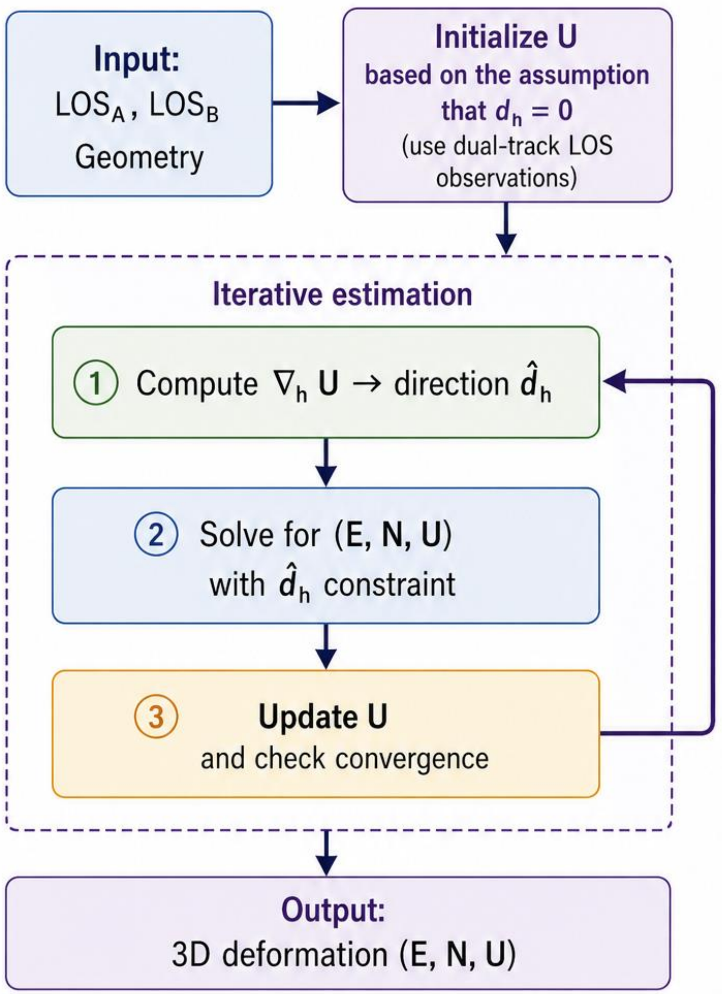
InSAR LOS Decomposition:
How to use the Gradient–Direction Constraint

$$\begin{bmatrix} d_a \\ d_d \end{bmatrix} = \begin{bmatrix} \cos(\theta_a) & \sin(\theta_a)\sin(\alpha_a - \gamma) \\ \cos(\theta_d) & \sin(\theta_d)\sin(\alpha_d - \gamma) \end{bmatrix} \begin{bmatrix} d_u \\ d_\gamma \end{bmatrix}$$

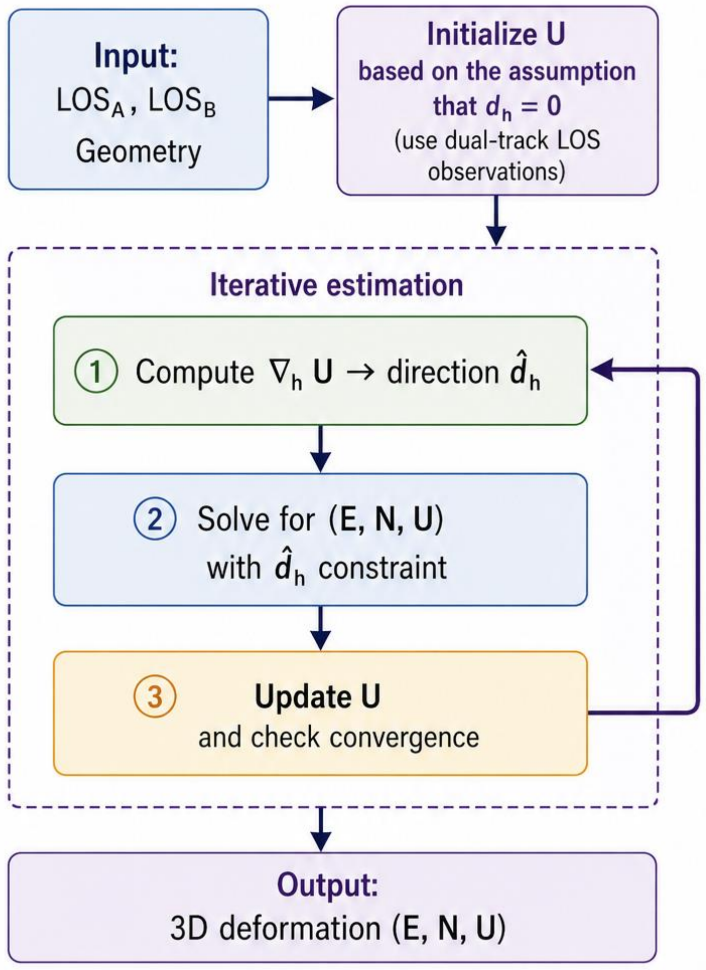
$$\gamma = \arctan\left(\frac{\partial d_u/\partial n}{\partial d_u/\partial e}\right)$$



Iterative approach



InSAR LOS Decomposition:
How to use the Gradient–Direction Constraint



When the estimated deformation direction approaches the north direction, the decomposition problem remains **ill-conditioned** and becomes highly **sensitive to measurement noise.**

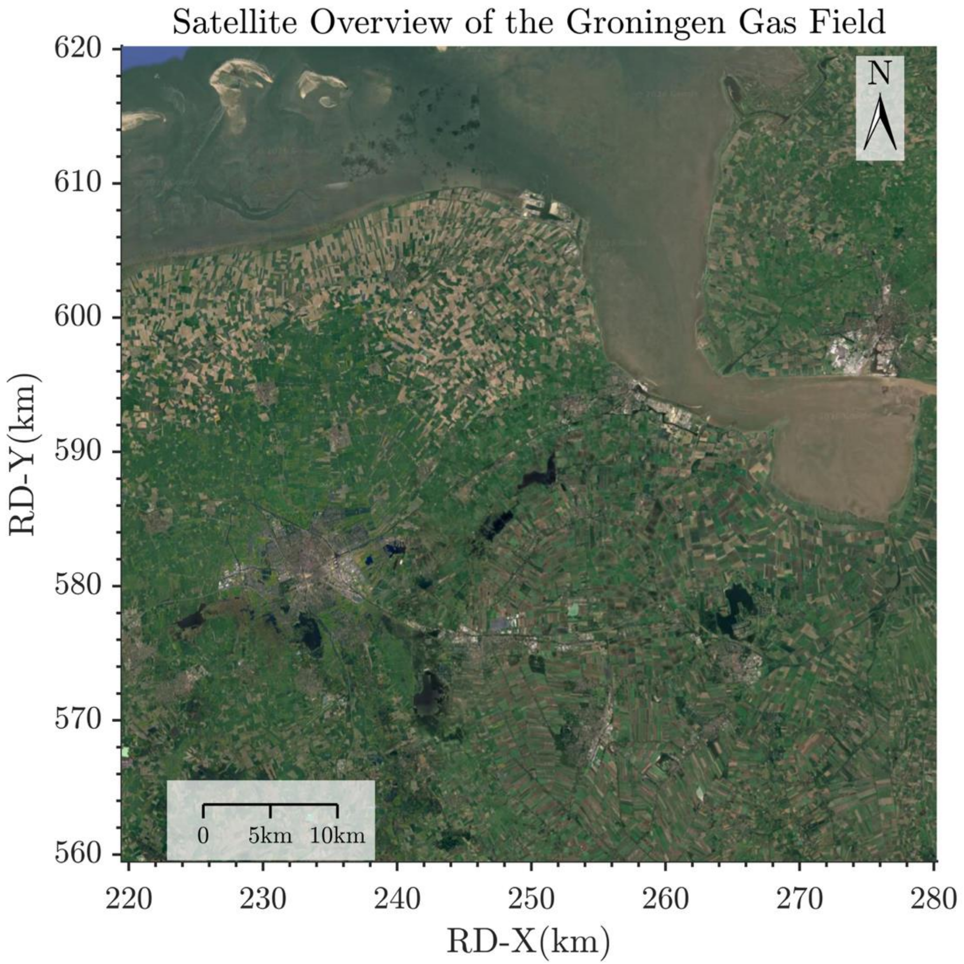
We need more restrict constrain then:

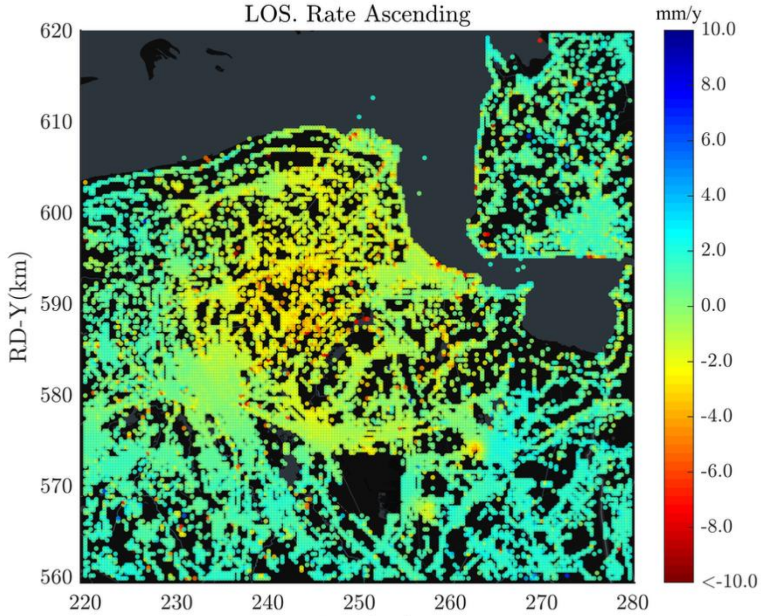
$$d_\gamma = K \arctan\left(\frac{\partial d_u / \partial n}{\partial d_u / \partial e}\right)$$

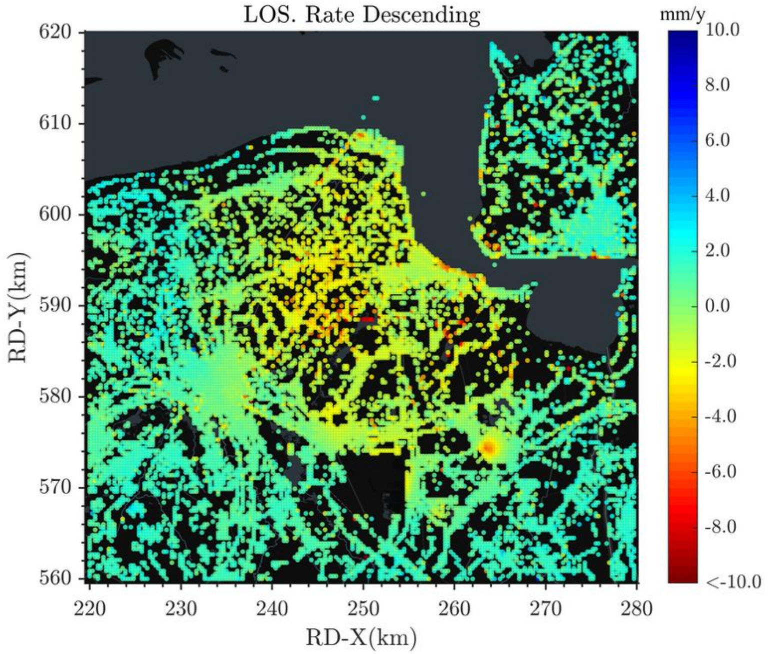
K is estimated in adjacent areas and then is interpolated to the problematic areas

InSAR LOS Decomposition: Case study

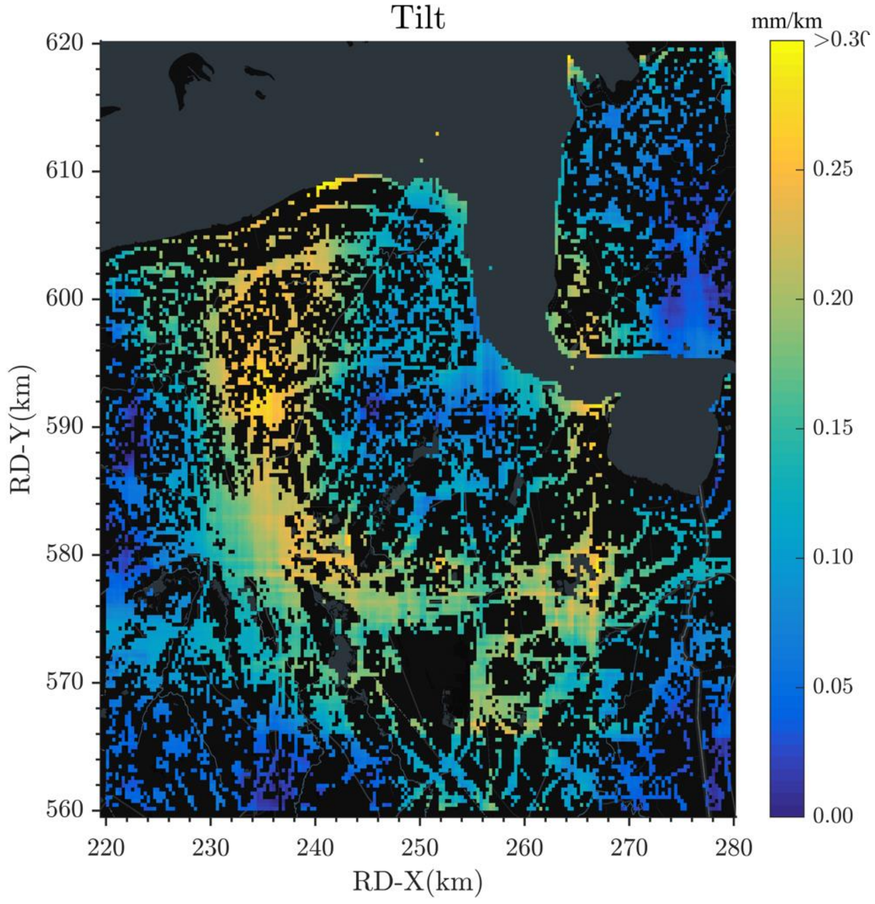
Groningen Gas Field (open access GPS and InSAR data available)

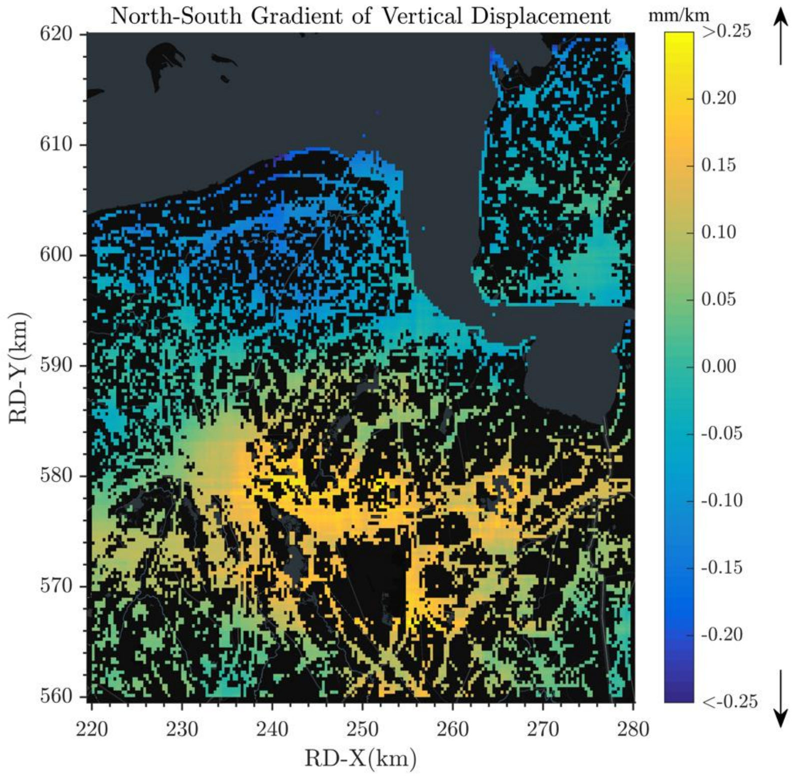


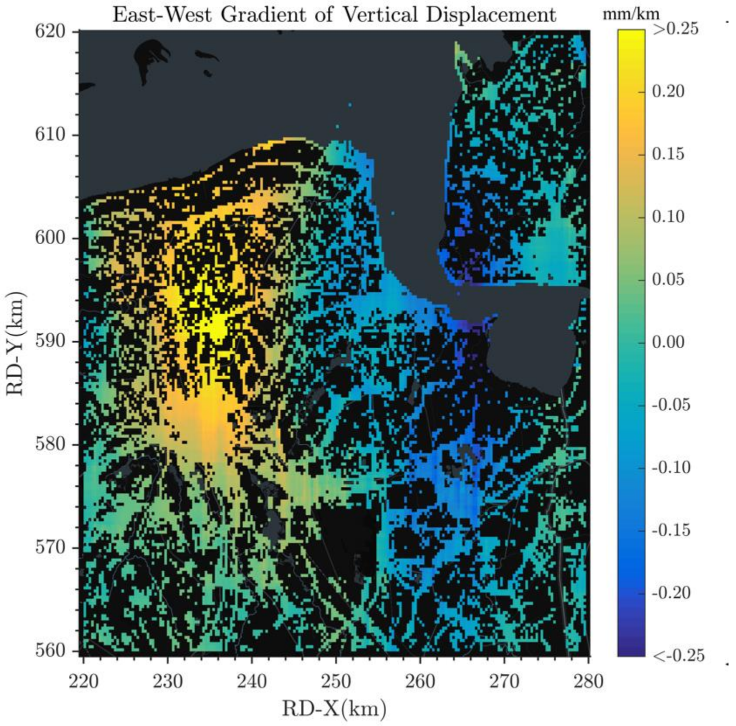




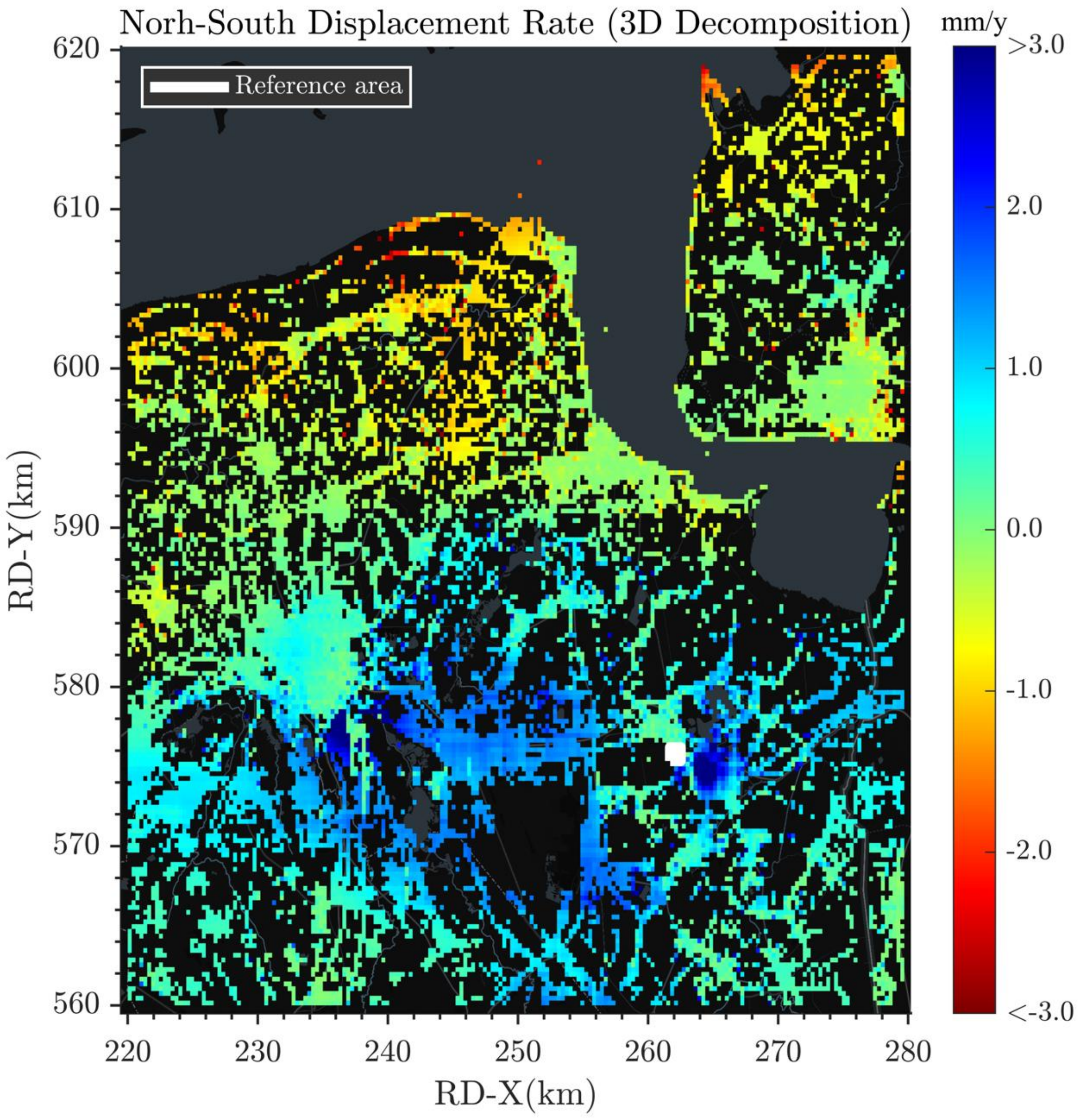
InSAR LOS Decomposition: Case study

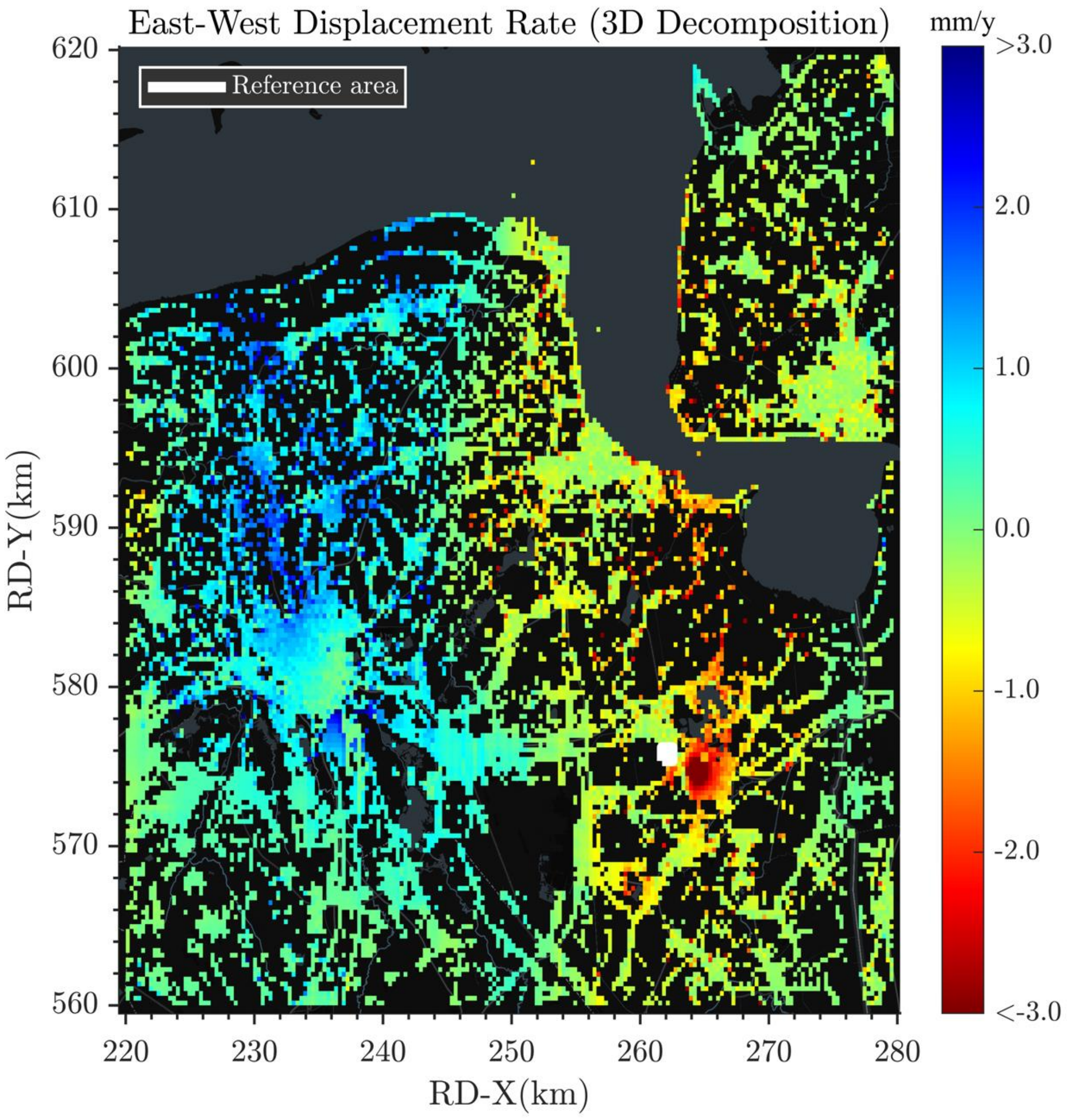




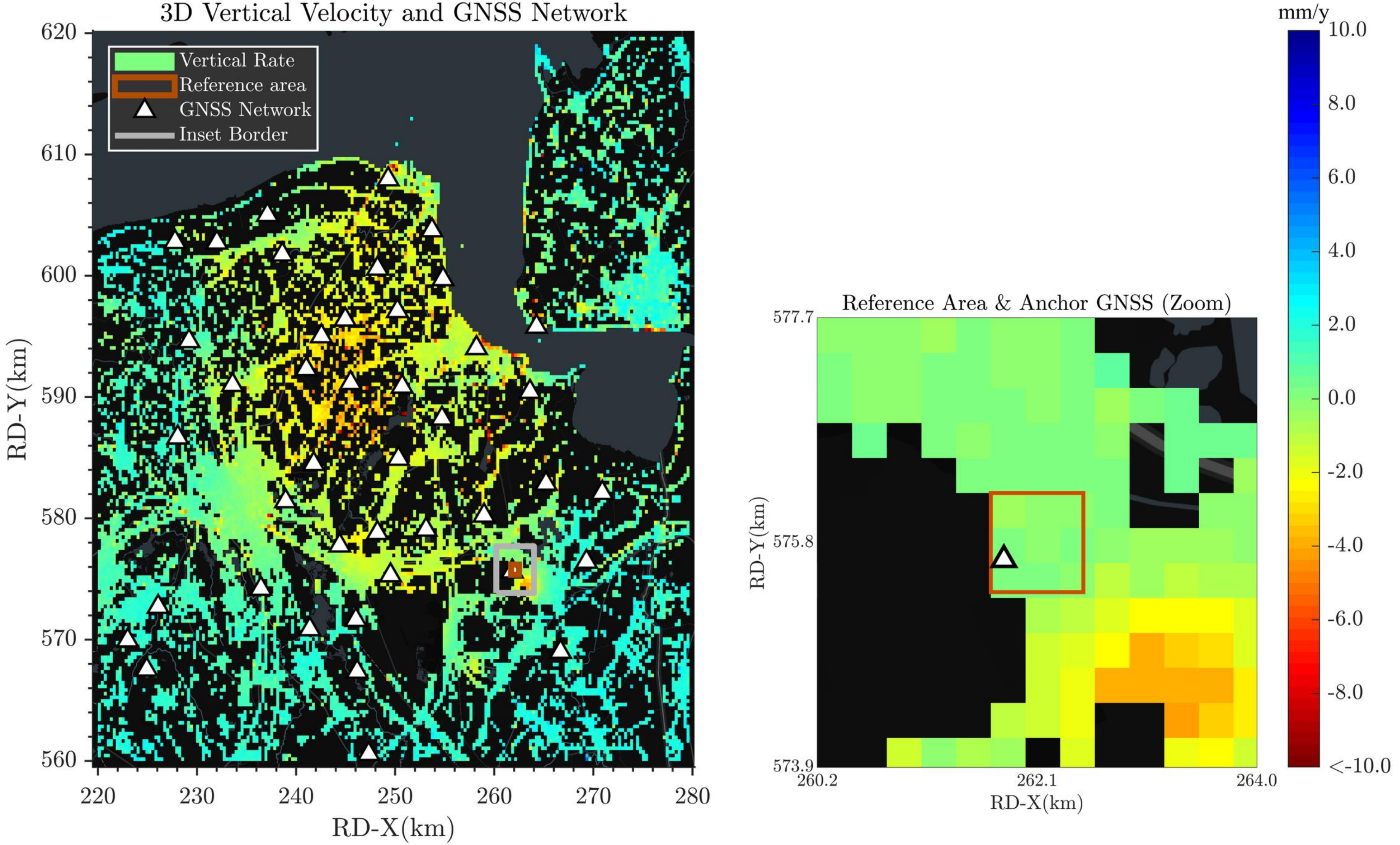


InSAR LOS Decomposition: Case study

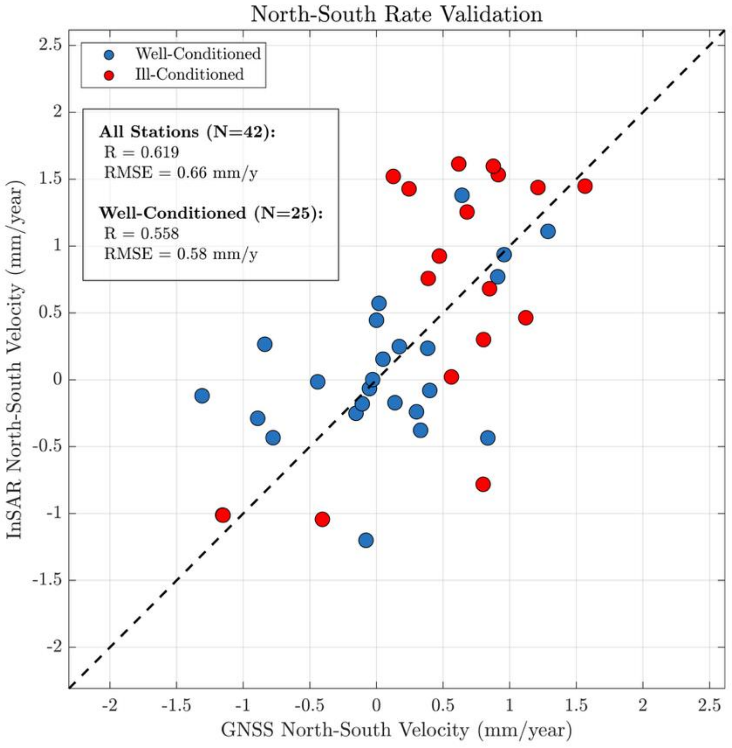


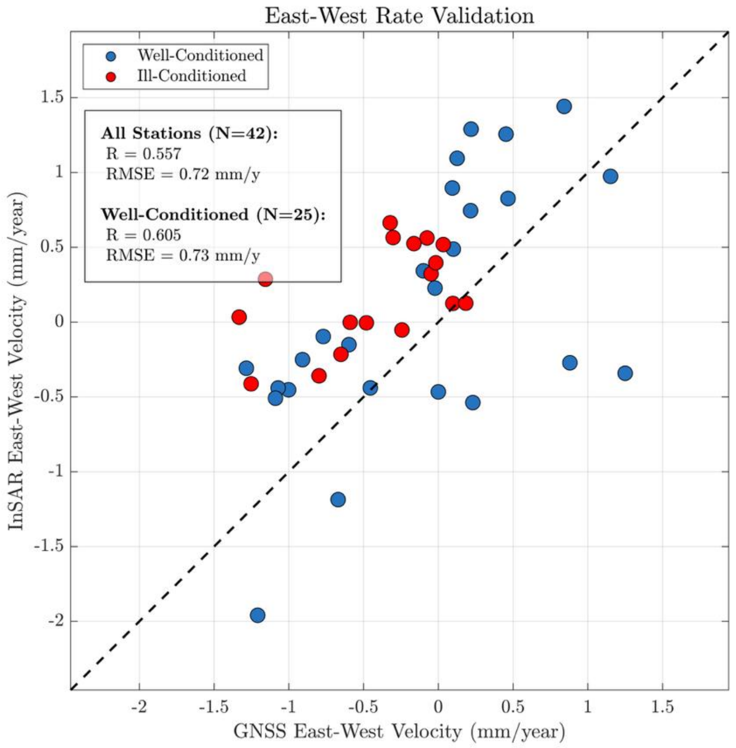


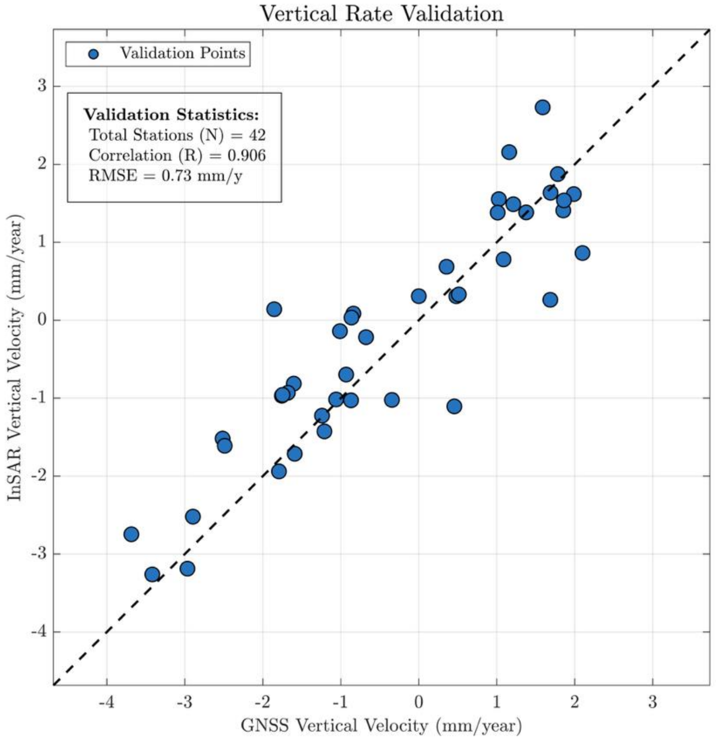
InSAR LOS Decomposition: Case study



InSAR LOS Decomposition: Case study

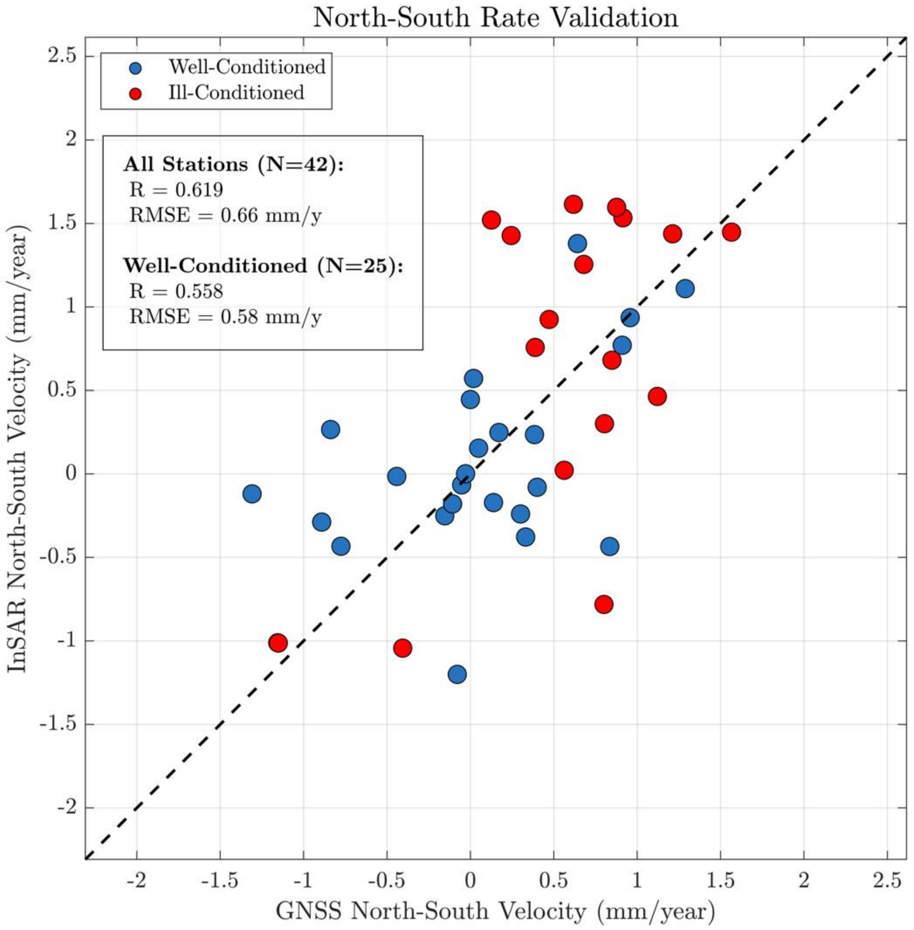




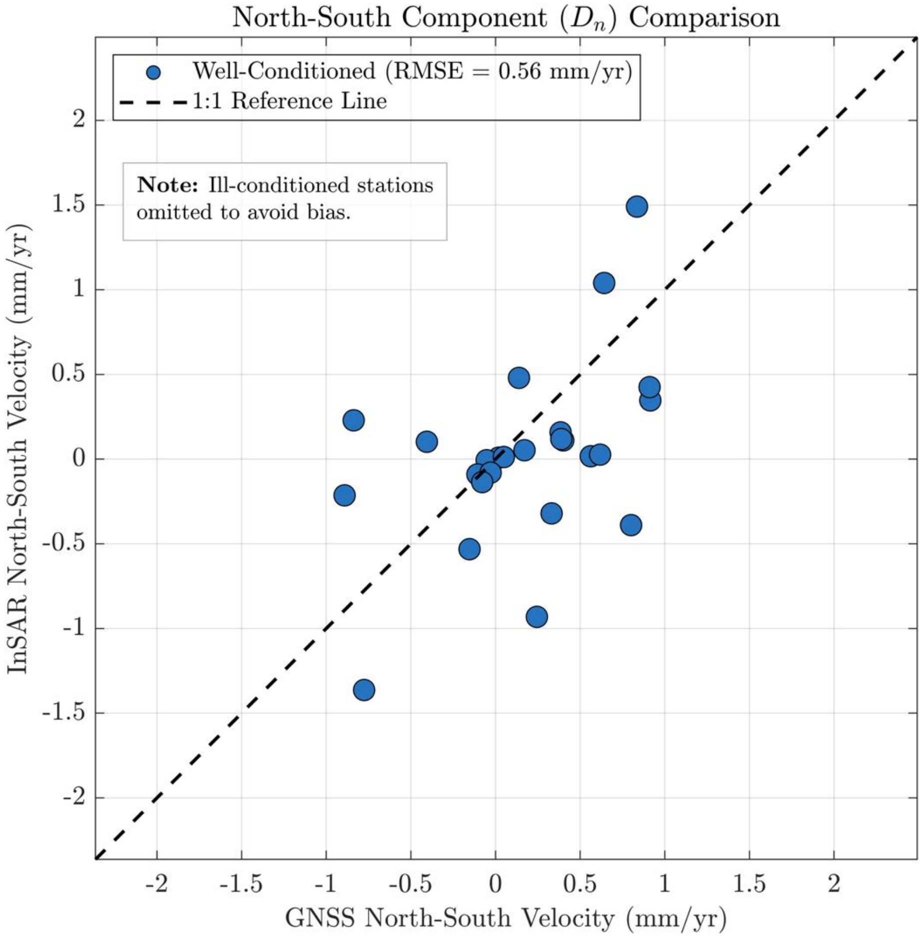


InSAR LOS Decomposition: Case study

Direction from our method

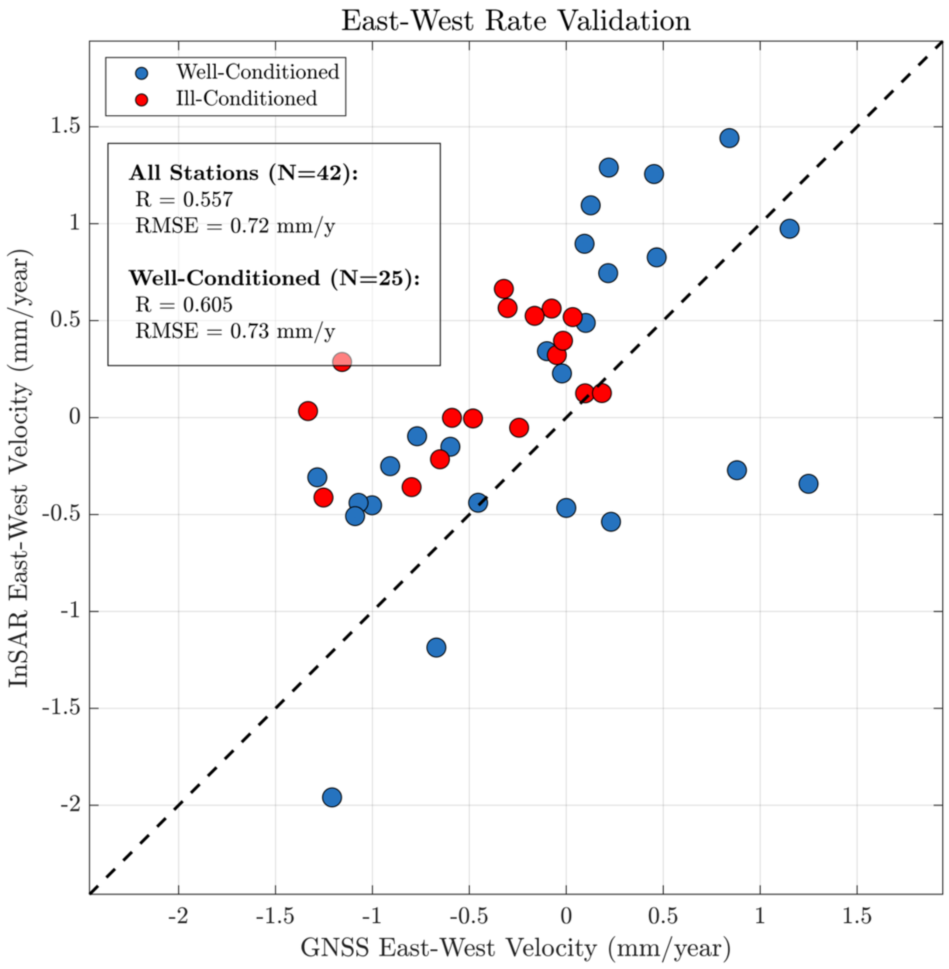


Direction from GPS

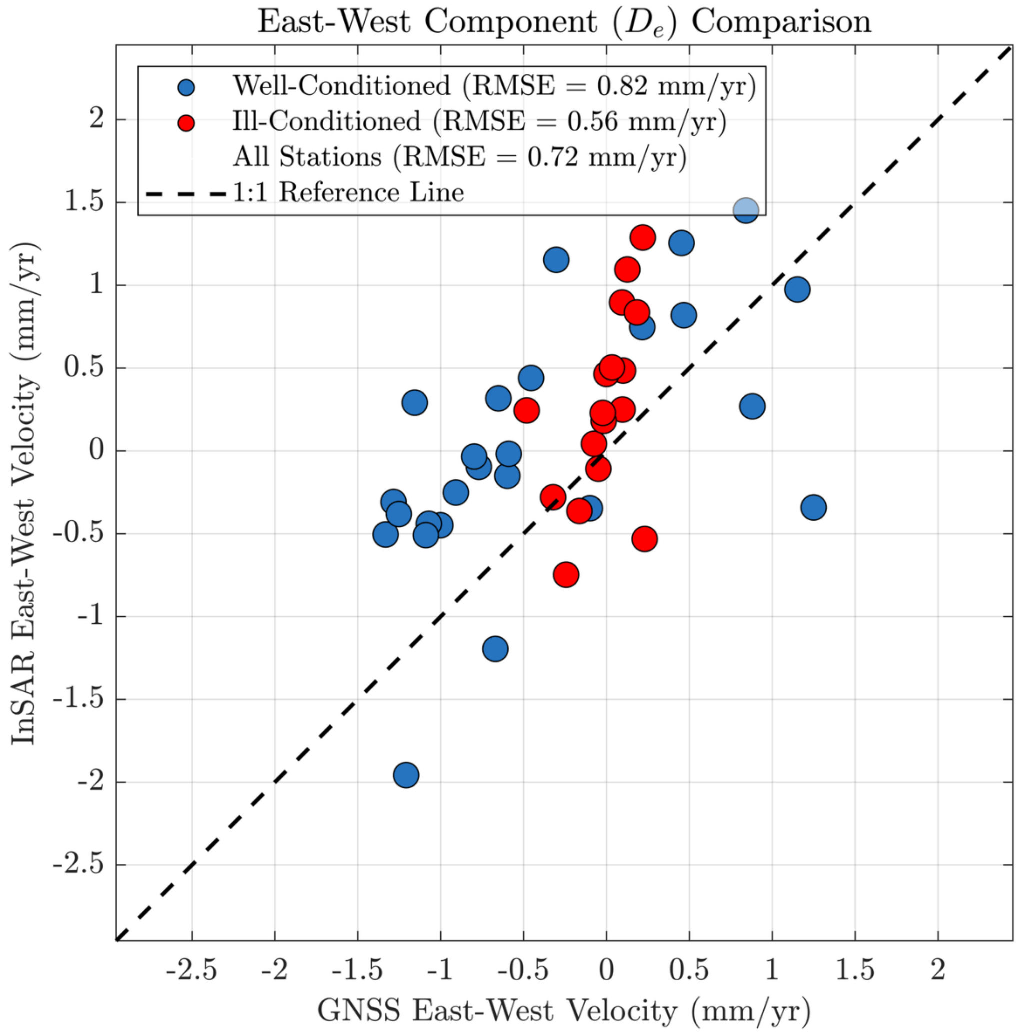


InSAR LOS Decomposition: Case study

Direction from our method



Direction from GPS



Decadal Assessment of Land Subsidence Hazards in Iran's Megacities Using Sentinel-1 Time-Series

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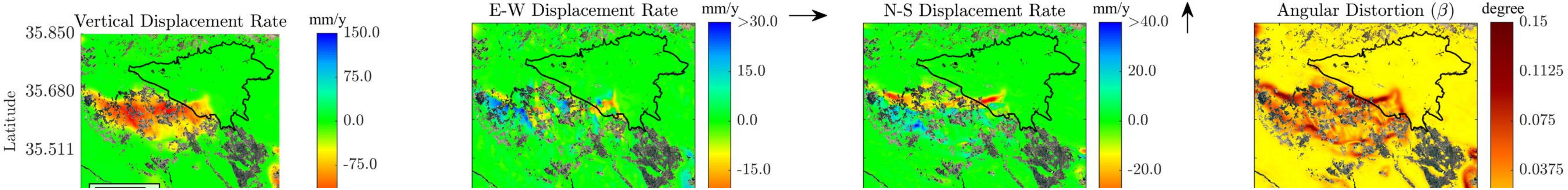
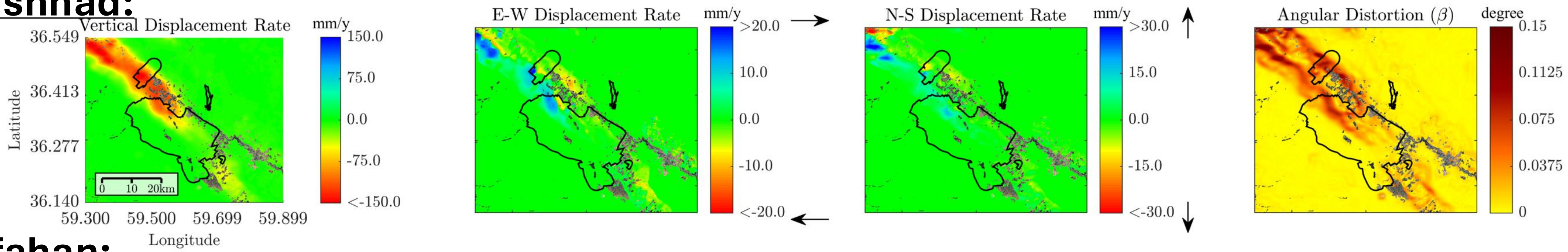
Tehran:



Mashhad:



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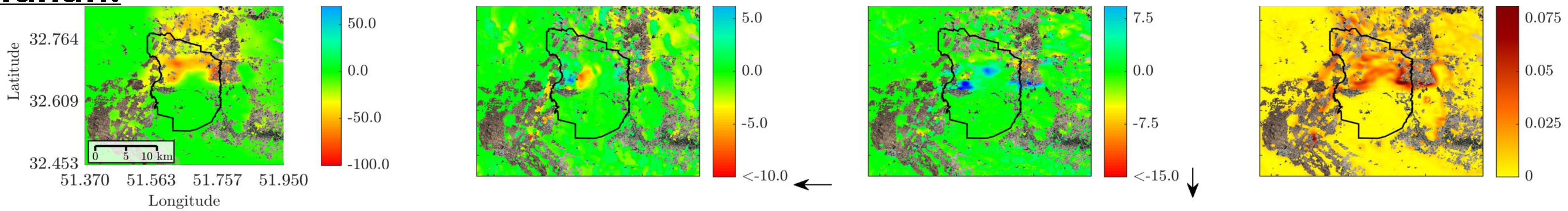

Take-Home Message

✓ Exact for Mogi sources

✓ Approximately valid for many realistic subsidence systems

✓ Enables retrieval of the north component from dual-track InSAR

✓ Most reliable for deep, smooth, volumetric deformation sources

⚠**Use with caution and validate whenever possible, particularly in regions with complex or mixed deformation mechanisms.**